ESSAYS IN LAW AND ECONOMICS

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Joshua Charles Teitelbaum

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ESSAYS IN LAW AND ECONOMICS

Joshua Charles Teitelbaum, Ph.D.

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The dissertation consists of three essays in law and economics. The first chapter compares the efficiency of negligence and strict liability in unilateral accident cases when the injurer faces ambiguity about accident risk. It generalizes the standard accident model to allow for ambiguity by assuming the injurer is a Choquet expected utility maximizer and representing the injurer's beliefs about accident risk with a neo-additive capacity. The central result is that neither strict liability nor negligence is generally efficient in the presence of ambiguity. A key implication of the results is that negligence is more robust to ambiguity, which may help explain why negligence is the general basis for accident liability under modern Anglo-American tort law.

The second chapter examines how different allocation rules influence the risk that putative class members will opt out of a mass tort class action. It analyzes a two-stage model of class action formation. The main result is that the class will be asymptotically stable if the net recovery will be allocated pro rata by expected claim values, but may not be asymptotically stable if the net recovery will be shared equally or allocated pro rata by damage claims. Other results explore how the shape of the distribution of the plaintiffs' damage claims, the scale benefits of the class action, and the plaintiffs' probability of prevailing and bargaining power in settlement negotiations influence the stability of the class.

The third chapter offers a model of analogical legal reasoning. Under the model, the outcome in the case at hand is a weighted average of the outcomes of prior cases, where the weights are a function of fact similarity and precedential authority. The main theoretical result is an axiomatization of similarity-weighted averaging

with an exponential similarity function based on a quasimetric. The chapter also investigates whether the analogical model provides a better fit than a rule-based model (represented by a fractional polynomial) to the reported decisions by federal judges in U.S. maritime salvage cases from 1880 to 2007. The principal conclusion of the empirical analysis is that the rule-based model fits the data better than the analogical model.

BIOGRAPHICAL SKETCH

Joshua Charles Teitelbaum was raised in Baltimore, Maryland. He is a graduate of Baltimore City College High School. He holds a Bachelor of Arts magna cum laude with Highest Honors in Economics from Williams College and a Juris Doctor cum laude from Harvard Law School.

To Vanessa, Katherine, and David

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PREFACE

The dissertation consists of three essays in law and economics. The essays are connected by two themes. The first theme, which connects the first and second chapters, is economics and torts. The second theme, which connects the first and third chapters, is decision theory and the law.

The first chapter of the dissertation compares the efficiency of negligence and strict liability in unilateral accident cases when the injurer faces ambiguity about accident risk. (Negligence and strict liability are the principal liability rules in Anglo-American tort law. In unilateral accidents, the injurer, but not the victim, can take care to reduce expected accident losses, and a liability rule is efficient if it induces the injurer to exercise optimal care.) The standard unilateral accident model is based on the expected utility framework and represents the injurer's beliefs about accident risk with a probability distribution. As a result, it does not allow for Knightian uncertainty, or ambiguity, with respect to accident risk and cannot accommodate different attitudes toward to ambiguity, including optimism and pessimism. The chapter generalizes the standard model to allow for ambiguity by assuming the injurer is a Choquet expected utility maximizer and representing the injurer's beliefs about accident risk with a special type of nonadditive probability called a neo-additive capacity. The central result of the model is that neither strict liability nor negligence is generally efficient in the presence of ambiguity. This is in contrast with the standard result that both strict liability and negligence are efficient. Comparative statics analysis suggests that the injurer's level of care decreases (increases) with ambiguity if he is optimistic (pessimistic) and decreases (increases) with his degree of optimism (pessimism). A key implication of the results is that negligence is more robust to ambiguity. This suggests that negligence may be superior to strict liability in unilateral accident cases. It also may help explain why negligence is the general basis for accident liability under modern Anglo-American tort law. Finally, the chapter designs and demonstrates the efficiency of an "ambiguity adjusted" liability rule.

The second chapter examines how different allocation rules influence the risk that putative class members will opt out of a mass tort class action. It analyzes a two-stage model of class action formation in which a single defendant faces multiple plaintiffs with heterogeneous damage claims. A global class action is certified at the outset. In stage 1, the plaintiffs play a coalition formation game in which each plaintiff simultaneously announces whether it will remain in the class or opt out. Stage 1 is modeled as a noncooperative game in partition function form. The global class is stable if the strategy profile in which all plaintiffs remain in the class constitutes a pure strategy Nash equilibrium of the game. In stage 2, the class action and any individual actions by opt-out plaintiffs are litigated or settled. Stage 2 is modeled in the divergent expectations tradition and assumes that if the parties settle their dispute they divide the joint surplus from settlement according to the asymmetric Nash bargaining solution. The main result of the model is that the class is asymptotically stable (i.e., the probability that it is stable converges to one as the class becomes arbitrarily large) if the net recovery of the class will be allocated pro rata in accordance with its members' outside options (their expected claim values), but that the class may not be asymptotically stable if the net recovery will be shared equally or allocated pro rata in accordance with the members' damage claims. The analysis suggests that the shape of the distribution of the plaintiffs' damage claims is a key determinant of class stability under equal sharing. It also suggests that the scale benefits of the class action and the plaintiffs' probability of prevailing at trial and bargaining power in settlement negotiations are important determinants of class stability. Monte Carlo simulations of the model compare the relative stability of each allocation rule. The results of the analysis suggest criteria to attorneys and courts for structuring and approving efficient allocations plans in mass tort class actions and for evaluating the "superiority" requirement for class certification in mass tort cases, which requires that a class action must be superior to other available methods for fairly and efficiently adjudicating the controversy.

The third chapter offers a model of analogical legal reasoning and examines whether the analogical model has more explanatory power than a rule-based model. The use of analogical reasoning in law is a central topic in the jurisprudence literature; commentators lament, however, that it is infrequently described with any rigor or care. Similarly, although the legal model of judicial decision making (which posits that judges decide cases solely by application of legal doctrine) is a leading paradigm in the literature, the lack of a formal model has led critics to argue that the legal model suffers from theoretical and empirical indeterminacy. The chapter contributions to the literature in two ways. First, it uses the apparatus of case-based decision theory to build a formal model of analogical legal reasoning. Under the model, the outcome in the case at hand is a weighted average of the outcomes of prior cases, where the weights are a function of the fact similarity and precedential authority of the prior cases. The main theoretical result is an axiomatization of similarity-weighted averaging with an exponential similarity function based on a quasimetric. The chapter's second contribution is an empirical investigation of whether the analogical model provides a better fit than a rule based-model (formally represented by a fractional polynomial) to the reported decisions by federal judges in U.S. maritime salvage cases from 1880 to 2007. The principal conclusion of the empirical analysis is that the rule-based model fits the data better than the analogical model. The chapter also presents a regression tree analysis of the maritime salvage cases as a supplement to the main empirical analysis. Finally, it discusses the implications and limitations of the empirical analysis.

CHAPTER 1

A UNILATERAL ACCIDENT MODEL UNDER AMBIGUITY

Fathered by Coase (1960), the economic analysis of tort law lies at the foundation of modern law and economics. The workhorse of tort law and economics is the basic accident model, which was first formalized by Brown (1973) and later expounded by Shavell (1987) and Landes and Posner (1987).[1] The basic accident model provides a framework for analyzing the effects of liability rules on agents' incentives to take care against accidents and, therefore, on the social costs of accidents. Consequently, it has been widely used to examine positive and normative questions about the efficiency and suitability of alternative liability rules.[2]

Standard formulations of the basic accident model assume that agents are expected utility maximizers in conformity with the decision theories of von Neumann and Morgenstern (1944), Savage (1954), or Anscombe and Aumann (1963). Under expected utility theory,[3] agents are assumed to make decisions under uncertainty as if they assign a probability distribution over the set of possible events and choose an act from the set of available acts that maximizes the expected value of a utility function with respect to such probability distribution. Due to its mathematical simplicity and normative appeal, as well as the explanatory power of many of its predictions, expected utility theory is the dominant framework for the analysis of individual decision making under uncertainty in economics.

Notwithstanding its primacy, there have been many challenges to expected utility theory as a positive decision theory. One of the most famous is the paradox

[1] A more recent comprehensive treatment of the basic accident model is contained in Miceli (1997).

[2] I adhere to the view that efficiency is the appropriate normative goal of the legal system. In defense of this view, see Kaplow and Shavell (2001, 2002b).

[3] I use the term "expected utility theory" broadly to encompass objective expected utility theory as formulated by von Neumann and Morgenstern (1944) as well as subjective expected utility theory as formulated by Savage (1954) and Anscome and Aumann (1963).

of Ellsberg (1961). One version of the Ellsberg paradox goes as follows. There are two urns. Urn I contains 50 red balls and 50 black balls. Urn II contains 100 red and black balls in an unknown proportion. Subjects engage in two gambles. In gamble A, subjects receive \$100 if they draw a red ball and nothing if they draw a black ball. In gamble B, subjects receive \$100 if they draw a black ball and nothing if they draw a red ball. Before each gamble, subjects choose the urn from which they prefer to draw the ball. Most subjects in this situation choose to draw from urn I in both gambles (see Becker and Brownson 1964). This result, however, is paradoxical to expected utility theory, for it would imply that the assigned probability of drawing a red ball from urn II is less than one half in gamble A and greater than one half in gamble B. In other words, these subjects are not acting as if they assigned probabilities to uncertain events and, therefore, expected utility theory cannot explain their choices.

The Ellsberg paradox highlights the significance of *ambiguity* for individual decision making. Knight (1921) made a distinction between *risk*—uncertain events with specified probabilities—and *uncertainty*—uncertain events with unspecified or ambiguous probabilities. The approach of expected utility theory, however, obviates this distinction. As a result, models based on the expected utility framework, including the basic accident model, do not allow for Knightian uncertainty, which has come to be known in the literature as ambiguity, and therefore cannot capture different attitudes toward or reactions to ambiguity, including ambiguity loving, or *optimism*, and ambiguity aversion, or *pessimism*.

Psychology research suggests that people exhibit optimism and pessimism in the accident context. Optimism has been found to be robust with respect to a variety of accident risks (see, e.g., Weinstein 1980, 1989, 1999; Sunstein 1997; Jolls 1998). In the case of traffic accidents, for example, studies have found that while people's beliefs about societal accident risks are fairly accurate (see, e.g., Lichtenstein et al. 1978), people generally are optimistic with respect to the likelihood that they will cause or otherwise be involved in an accident (see Svenson 1981; Svenson et al. 1985; Finn and Bragg 1986; Matthews and Moran 1986; DeJoy 1989; McKenna et al. 1991; Guppy 1992). Pessimism tends to be displayed with respect to the risk of accidents that are *available*—for example, highly salient, perhaps due to media attention; dramatic or catastrophic in nature; intrinsically vivid, imaginable, or memorable; or technological in nature (see Sunstein 1997; Jolls 1998; Jolls et al. 1998; Gigerenzer 2005; see also Slovic et al. 1982; Covello and Johnson 1987; Viscusi and Magat 1987; Viscusi 1992). This research calls for the modification of the basic accident model to allow for ambiguity.

The Ellsberg paradox and subsequent experimental evidence of the importance of ambiguity attitudes for decisions[4] have inspired various alternatives to and generalizations of expected utility theory to accommodate ambiguity.[5] One of the most influential axiomatic generalizations of expected utility theory that accommodates ambiguity is *Choquet expected utility theory*, which was pioneered by Schmeidler (1989).[6] Under Choquet expected utility theory, agents' beliefs about the likelihood of uncertain events are represented with a non-additive probability called a *capacity*.[7] Agents are assumed to act so as to maximize the expected value of a utility function with respect to such capacity, which is calculated using the Choquet (1954) integral. The non-additivity of the capacity allows for different ambiguity attitudes. In particular, a concave (superadditive) capacity reflects optimism while

[4]For a more detailed discussion of the Ellsberg paradox and a survey of the related experimental evidence, see Camerer (1995).

[5]For a survey of alternatives to and generalizations of expected utility theory, including those that accomodate ambiguity, see Camerer and Weber (1992).

[6]Schmeidler (1989) and Gilboa (1987) axiomatize Choquet expected utility in the Anscome and Aumann (1963) and Savage (1954) frameworks, respectively. An additional axiomatization of Choquet expected utility is provided by Sarin and Wakker (1992).

[7]Expected utility is a special case of Choquet expected utility in which the capacity is additive (that is, a probability). A capacity μ is additive if $\mu(E) + \mu(F) = \mu(E \cup F)$ for all mutually exclusive events E and F.

a convex (subadditive) capacity reflects pessimism (see Schmeidler 1989; Wakker 2001).[8] Choquet expected utility with a convex capacity, for instance, can capture pessimism as exemplified by the Ellsberg paradox.

This chapter presents a unilateral accident model under ambiguity.[9] Specifically, it generalizes the basic unilateral accident model to allow for ambiguity by assuming the injurer is a Choquet expected utility maximizer and representing the injurer's beliefs about accident risk with a special type of capacity called a *neo-additive capacity*, which was introduced by Chateauneuf et al. (2007). Choquet expected utility with a neo-additive capacity is the simplest generalization of expected utility that can accommodate optimistic and pessimistic reactions to ambiguity. It assumes that an agent makes decisions under uncertainty as if he believes, with incomplete confidence, that a specified probability distribution describes the likelihood of uncertain events and chooses an act from the set of available acts that maximizes a weighted sum of the minimum utility, the maximum utility, and the expected utility with respect to such probability distribution. The parameters of the model allow us to measure the injurer's degree of ambiguity, which is the complement of his degree of confidence, and his degrees of optimism and pessimism, which determine the respective weights assigned to the maximum utility and the minimum utility. As a result, we can perform comparative statics on changes in optimism, pessimism, and ambiguity.[10]

I show that in the basic unilateral accident setting neither strict liability nor negligence is generally efficient in the presence of ambiguity. In particular, I show

[8] A capcity μ is convex if $\mu(E)+\mu(F) \leq \mu(E \cup F)$ for all mutually exclusive events E and F. It is concave if the reverse inequality holds.

[9] In unilateral accidents, the injurer, but not the victim, can take care to reduce expected accident losses.

[10] Closely related to Choquet expected utility with a neo-additive capacity is α-maxmin expected utility with multiple priors (Ghirardato et al. 2004). Under this approach, ambiguity is represented by a set of probability distributions, and optimism and pessimism correspond to the respective weights applied to the maximum and minimum expected utility over the set of probability distributions.

that (i) in the case of fixed accident losses (when the injurer's level of care does not affect the magnitude of accident losses), the injurer will exercise too little care under strict liability and may exercise too little care under negligence and (ii) in the case of variable accident losses (when the injurer's level of care does affect the magnitude of accident losses), the injurer may exercise too little or too much care under strict liability and may exercise too little care under negligence. In addition, I find that, in general, the injurer's level of care (i) decreases with his degree of optimism and increases with his degree of pessimism and (ii) decreases with ambiguity if he is optimistic and increases with ambiguity if he is pessimistic. The results are in contrast with the standard results of the basic unilateral accident model, namely that, in both cases on accident losses, the injurer will take optimal care under strict liability and negligence. The results suggest that negligence is more robust to ambiguity and, therefore, may be superior to strict liability in unilateral accident cases. Finally, I design and demonstrate the efficiency of an *ambiguity adjusted* liability rule.

This chapter contributes to strands of the law and economics and the applied decision theory literatures. Within the law and economics literature, this chapter is the first to adopt the Choquet expected utility framework to incorporate ambiguity with respect to accident risk into the basic accident model.[11] As such, it contributes to the well-established literature on the economics of tort law[12] and to the burgeoning behavioral law and economics literature.[13] Within the applied de-

[11]There are many papers that study the effects of uncertainty with respect to other aspects of the basic accident model. See *infra* footnotes 22-23. Shavell (1992) considers a situation in which agents face uncertainty about accident risk, but he examines the incentives that alternative liability rules create for injurers to obtain information about accident risk and whether these incentives are socially optimal. There are a limited number of papers that consider the effects of ambiguity on the economic analysis of other areas of law, for example, taxation (Jolls 1998; Chorvat 2002) and the criminal process (Segal and Stein 2006).

[12]Surveys of this literature are contained in Bouckaert and De Geest (2000, part 3), Kaplow and Shavell (2002a, pp. 1667-1682), and Mattiacci and Parisi (2005).

[13]Sunstein (1997) and Jolls, Sunstein, and Thaler (1998) were early calls for the modification of standard law and economics models to reflect advances in behavioral economics and decision

cision theory literature, this chapter adds to the growing number of applications of Choquet expected utility theory to accommodate ambiguity,[14] including applications that use neo-additive capacities to represent beliefs (see, e.g., Schipper 2005; Chateauneuf et al. 2007; Eichberger et al. 2008; Ford et al. 2008; Eichberger and Kelsey 2009).

Most closely related to this chapter are works by Posner (2003), Eide (2005, 2007), and Bigus (2006). Posner (2003) introduces optimism about low probability accidents into the basic unilateral accident model by assuming agents know the probability of an accident when it is above some threshold but treat accident probabilities below the threshold as though they were zero. For the case of fixed accident losses, he finds that, under both strict liability and negligence, agents might take too much or too little care for sufficiently high levels of optimism and will take optimal care for sufficiently low levels of optimism. Posner also analyzes the case of variable activity levels and briefly discusses bilateral accidents, neither of which I address in this chapter. However, he does not consider the case of unilateral accidents with variable accident losses, which I do. Eide (2005, 2007) and Bigus (2006) analyze the basic accident model under rank dependent expected utility theory (Quiggin 1982, 1993) and prospect theory (Kahneman and Tversky 1979), respectively.[15] For the case of unilateral accidents, Eide finds that under strict liability the injurer may take too much or too little care depending on the slope of the probability weighting function and that under negligence the injurer may take too little care if he substantially underweights the probability of an accident. Bigus

theory. Sunstein (2000) and Parisi and Smith (2005) are recent collections of behavioral law and economics papers.

[14] For a survey of this literature, see Mukerji and Tallon (2004).

[15] Rank dependent expected utility theory is a special case of Choquet expected utility theory in which the agent's capacity is an increasing probability weighting function (see Wakker 1990; Hong and Wakker 1996). Prospect theory is an alternative decision theory that is not directly related to Choquet expected utility theory. However, cumulative prospect theory (Tversky and Kaneman 1992) is a generalization of Choquet expected utility theory that permits different treatment of gains and losses (see Tversky and Wakker 1995).

finds that the injurer will take too little care under strict liability and may take too little care under negligence depending on the slope of the probability weighting function. These findings are consistent with the results of this chapter. Eide and Bigus also study bilateral accidents and vague standards of due care, respectively. Neither chapter, however, distinguishes the cases of fixed and variable accident losses or performs comparative statics.

The remainder of the chapter is organized as follows. Section 1.1 presents the model. It describes the basic setup and explains how ambiguity about accident risk is modeled by Choquet expected utility with a neo-additive capacity. Section 1.2 states the results of the model in the absence of ambiguity, which correspond to the standard results of the basic unilateral accident model, and derives the results of the model in the presence of ambiguity. Section 1.3 develops a simple numerical example to illustrate the model's results. Section 1.4 discusses certain implications of the model and designs an ambiguity adjusted liability rule that is efficient in the presence of ambiguity. Section 1.5 contains concluding remarks and suggests directions for future research. A formal description of the Choquet expected utility framework is set forth in the Appendix.

1.1 THE MODEL

1.1.1 Basic Setup

The model is based on the basic unilateral accident model of Shavell (1987). There are two agents—an injurer and a victim—and a numeraire good—income—in terms of which all payoffs and costs are defined. Both agents are risk neutral and their Bernoulli utility of income is equal to income. The agents are strangers and not parties to any contract or market transaction, and transaction costs are sufficiently high to preclude Coasian bargaining.

Each agent engages in a risky activity from which he receives a payoff. For example, the injurer could be driving a car and the victim could be a pedestrian. Let $k > 0$ denote the payoff to the injurer from engaging in his activity and normalize the victim's payoff to be zero. The injurer, but not the victim, has the ability to choose a level of care, expressed in terms of its cost, to exercise when engaging in his activity. Let $c \geq 0$ denote the level of care exercised by the injurer. An accident involving the injurer and the victim occurs with probability $\pi \in (0, 1]$. In the event of an accident, the victim incurs accident losses $l > 0$. Hence, expected accident losses are $L = \pi l$.

I consider two cases on accident losses: *fixed accident losses* and *variable accident losses*. In the case of fixed accident losses, the injurer can take care to reduce the probability of an accident, but the magnitude of accident losses is fixed. Thus, expected accident losses are $L(c) = \pi(c)l$. In the case of variable accident losses, the injurer can take care to reduce the probability of an accident and the magnitude of accident losses, and therefore expected accident losses are $L(c) = \pi(c)l(c)$. In both cases, I assume $\pi(c)$ is twice continuously differentiable, strictly decreasing, and strictly convex—$\pi'(c) < 0$ and $\pi''(c) > 0$. In the case of variable accident losses, I further assume $l(c)$ is twice continuously differentiable, strictly decreasing, and strictly convex—$l'(c) < 0$ and $l''(c) > 0$.

Whether the victim receives compensation from the injurer for accident losses depends on the applicable liability rule. I consider three liabilities rules: (i) *no liability*, under which the victim receives no compensation from the injurer, regardless of the level of care exercised by the injurer; (ii) *strict liability*, under which the victim receives full compensation for his accident losses from the injurer, regardless of the injurer's level of care; and (iii) *negligence*, under which the victim receives full compensation for his accident losses if the injurer fails to meet the applicable standard of due care, denoted $\bar{c}$. In modern Anglo-American law, negligence is the general basis for liability in cases of accidents among non-contracting parties or strangers. Strict liability applies only in certain accident cases, including cases in which the injurer engages in an abnormally dangerous activity or manufactures a defective product, certain nuisance and trespass cases, and cases involving certain environmental harms (see Dobbs 2001).

1.1.2 Modeling Ambiguity

In departure from the basic unilateral accident model, I assume the injurer faces ambiguity with respect to accident risk. To incorporate ambiguity into the model, I assume the injurer is a Choquet expected utility maximizer whose beliefs about accident risk may be represented with a neo-additive capacity ν based on π.[16] For simplicity, I assume the victim is an expected utility maximizer.

Formally, I assume the injurer's belief about the likelihood of an accident is given by $\nu(\pi) = \delta(1-\alpha) + (1-\delta)\pi(c)$, where $\delta, \alpha \in [0,1]$ and we normalize $\nu(0) = 0$ and $\nu(1) = 1$. Similarly, the injurer's belief about the likelihood of no accident is given by $\nu(1-\pi) = \delta(1-\alpha) + (1-\delta)(1-\pi(c))$. Note that, in general, the injurer's beliefs are non-additive: $\nu(\pi) + \nu(1-\pi) \neq 1$ unless $\delta = 0$ or $\alpha = \frac{1}{2}$. Given his beliefs, the injurer's Choquet expected utility of exercising level of care c under a rule of no liability or strict liability is

$$V_\pi(c) = \delta\alpha m(c) + \delta(1-\alpha)M(c) + (1-\delta)E_\pi(c) \tag{1.1}$$

where $m(c)$, $M(c)$, and $E_\pi(c)$ denote the minimum utility, the maximum utility, and the expected utility with respect to π, respectively, of exercising level of care c given the applicable liability rule. Under a negligence rule, the injurer effectively faces no liability if he satisfies the standard of due care ($c \geq \bar{c}$) and faces strict li-

[16] More precisely, ν is based on the probability distribution $\{\pi, 1-\pi\}$. To simplify the notation, however, I occassionally let π stand for the probability distribution $\{\pi, 1-\pi\}$.

ability otherwise ($c < \bar{c}$). Thus, the injurer's Choquet expected utility of exercising level of care c under a negligence rule is $\begin{cases} V_\pi(c) \text{ under no liability} & \text{if } c \geq \bar{c} \\ V_\pi(c) \text{ under strict liability} & \text{if } c < \bar{c} \end{cases}$.[17]

Note that because there are only two possible events—accident or no accident—and given the basic setup of the model, under each liability rule the minimum utility $m(c)$ is the outcome in the event of an accident, the maximum utility $M(c)$ is the outcome in the event of no accident, and the expected utility $E_\pi(c)$ is the expected outcome $\pi(c)m(c) + (1 - \pi(c))M(c)$. Accordingly, we can rewrite equation (1.1) as

$$V_\pi(c) = [\delta\alpha + (1-\delta)\pi(c)]m(c) + [\delta(1-\alpha) + (1-\delta)(1-\pi(c))]M(c). \tag{1.2}$$

From equation (1.2) we can see that in evaluating the Choquet expected utility of exercising level of care c, the injurer assigns weight $\delta\alpha + (1-\delta)\pi(c)$ to the accident outcome, $m(c)$, and weight $\delta(1-\alpha) + (1-\delta)(1-\pi(c))$ to the no accident outcome, $M(c)$. It is important to note that these weights are not subjective probabilities corresponding to the injurer's beliefs about accident risk but rather are decision weights generated by a neo-additive capacity based on π that represents his non-additive beliefs.[18] In particular, the weight assigned to the accident outcome, $\delta\alpha + (1-\delta)\pi(c)$, does not correspond to the injurer's belief about the likelihood of an accident, $\nu(\pi)$.[19]

Intuitively, representing the injurer's beliefs about accident risk with a neo-

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[17] Technical details underlying ν and V_π are supplied in the Appendix.

[18] For a detailed discussion of capacities and decision weights, see Sarin and Wakker (1998).

[19] In fact, they coincide only if there is no ambiguity ($\delta = 0$), the injurer is neither optimistic nor pessimistic ($\alpha = \pi$), or the injurer reacts to ambiguity with equal degrees of optimism and pessimism ($\alpha = \frac{1}{2}$). Note, however, that the weight assigned to the no accident outcome, $\delta(1-\alpha) + (1-\delta)(1-\pi(c))$, does correspond to the injurer's belief about the likelihood of no accident, $\nu(1-\pi)$, and that $\delta\alpha + (1-\delta)\pi(c) = 1 - \nu(1-\pi)$. The former reflects a general property of Choquet expected utility with a neo-additive capacity, namely that the weight assigned to the best outcome corresponds to the agent's capacity of the best outcome (see Eichberger and Kelsey 2009). The latter reflects the peculiar fact of the model that there are only two outcomes. It does not hold in the general case where there are more than two outcomes (see Eichberger and Kelsey 2009).

additive capacity based on π assumes the injurer believes the probability of an accident is π, but lacks confidence in this belief. The injurer's degree of confidence is measured by $(1 - \delta)$. It is the weight the injurer puts on π in his capacity and on E_π in his utility function. The *degree of ambiguity* is measured by δ. It represents the degree to which the injurer lacks confidence in π. The injurer reacts to ambiguity by overweighting either the outcome in the event of an accident, m, or the outcome in the event of no accident, M. If the injurer overweights the accident outcome, we say he is *pessimistic*. If he overweights the no accident outcome, we say he is *optimistic*. Which outcome the injurer overweights depends on the parameter α. If $\alpha > \pi$, the injurer overweights the accident outcome; if $\alpha < \pi$, the injurer overweights the no accident outcome.[20] Accordingly, we interpret α as the injurer's *degree of pessimism* and $1 - \alpha$ as his *degree of optimism*. Note that if there is no ambiguity ($\delta = 0$) or if the injurer is neither optimistic nor pessimistic ($\alpha = \pi$), then Choquet expected utility with respect to ν based on π reduces to expected utility with respect to π (i.e., $V_\pi = E_\pi$) and the model reduces to the standard unilateral accident model.[21]

In order to focus on the effects of ambiguity on the standard results of the basic unilateral accident model, I assume there is no uncertainty with respect to any other aspect of the model.[22] For example, I assume that the agents know the applicable legal standards and that the court accurately determines all relevant facts, including the probability of an accident, the magnitude and incidence of accident losses, and the agents' preferences and acts.[23] In addition, I abstract from

[20] To see this, note that $\delta\alpha + (1-\delta)\pi > \pi$ if and only if $\alpha > \pi$ and that $\delta(1-\alpha) + (1-\delta)(1-\pi) > 1 - \pi$ if and only if $\alpha < \pi$.

[21] To see that $V_\pi(c) = E_\pi(c)$ if there is no ambiguity, simply substitute $\delta = 0$ into equation (1). To see that $V_\pi(c) = E_\pi(c)$ if the injurer is neither optimistic nor pessimistic, substitute $\alpha = \pi$ into equation (2) and recall that $E_\pi(c) = \pi(c)m(c) + (1 - \pi(c))M(c)$.

[22] Shavell (1987) and Miceli (1997) provide textbook coverage of various models that introduce uncertainty with respect to other aspects of the basic accident model.

[23] Craswell and Calfee (1986) present an accident model in which defendants face uncertainty about the applicable legal standards. Shavell (1985) presents an accident model in which courts

other complexities that have been introduced in the literature, such as bilateral care, variable activity levels, bilateral harm, risk aversion, and the judgment proof problem.[24]

1.1.3 Remarks

Before turning to the results of the model, I conclude this section with a few general remarks regarding Choquet expected utility with a neo-additive capacity.

Capacities and ambiguity. A capacity can capture different ambiguity attitudes because it is non-additive. To illustrate, let us show how a convex neo-additive capacity can capture ambiguity aversion, or pessimism, as exemplified by a preference for urn I (the unambiguous urn) in both gambles in the Ellsberg paradox. Let $u(x)$ denote the utility of prize x and normalize $u(0) = 0$. Consider an agent who evaluates gambles according to expected utility and whose belief about the likelihood of drawing a red ball from urn II (the ambiguous urn) is given by a probability p. A preference for urn I in gamble A implies $\frac{1}{2}u(100) + \frac{1}{2}u(0) > pu(100) + (1-p)u(0)$, or $\frac{1}{2} > p$, and a preference for urn I in gamble B implies $\frac{1}{2}u(0) + \frac{1}{2}u(100) > pu(0) + (1-p)u(100)$, or $\frac{1}{2} > 1-p$. Combining these conditions we have $p + (1-p) < 1$, which contradicts the additivity of p. This illustrates the paradox. It also illustrates that ambiguity aversion is akin to subadditivity. Next consider an agent who evaluates gambles according to Cho-

face uncertainty about causation. Hylton (1990) presents an accident model in which courts are unable to determine accurately in every case whether the defendant acted negligently.

[24] I restrict attention to unilateral care because it is the primitive form of the basic accident model. In conformity with the basic accident model, I assume that the agents' activity levels are fixed and do not affect expected accident losses, that only the victim incurs accident losses, and that the agents are risk neutral. Shavell (1980) introduces the issue of the choice of activity level to the basic accident model. Leong (1989) and Arlen (1990, 1992) present models in which both the injurer and the victim incur accident losses. Shavell (1982) introduces risk aversion into the basic unilateral accident model. Shavell (1986) examines the judgment proof problem. For additional complexities that have been introduced in the literature, see generally Shavell (1987) and Miceli (1997).

quet expected utility and whose belief about the likelihood of a drawing a red ball from urn II is given by a convex neo-additive capacity μ based on p. In addition, assume the agent faces ambiguity ($\delta > 0$). Now a preference for urn I in gamble A implies $\frac{1}{2}u(100) + \frac{1}{2}u(0) > [\delta\alpha + (1-\delta)(1-p)]u(0) + [\delta(1-\alpha) + (1-\delta)p]u(100)$, or $\frac{1}{2} > \delta(1-\alpha) + (1-\delta)p = \mu(p)$, and a preference for urn I in gamble B implies $\frac{1}{2}u(0) + \frac{1}{2}u(100) > [\delta\alpha + (1-\delta)p]u(0) + [\delta(1-\alpha) + (1-\delta)(1-p)]u(100)$, or $\frac{1}{2} > \delta(1-\alpha) + (1-\delta)(1-p) = \mu(1-p)$. Combining these conditions we have $\mu(p) + \mu(1-p) < 1$, which is consistent with the convexity of μ.

Neo-additive capacities. A neo-additive capacity is a probability weighting function. In particular, it is a simple version of the familiar inverse-S shaped probability weighting function from cumulative prospect theory (Tversky and Kaneman 1992). Empirical studies indicate that individuals tend to overweight low probabilities and underweight high probabilities with the most pronounced misweighting near the extremes of the probability scale (see Gonzalez and Wu 1999). This systematic distortion of probabilities implies an inverse-S shaped probability weighting function. There is overwhelming evidence from parametric and non-parametric studies for the inverse-S shape (see Wakker 2001). A standard non-linear specification of an inverse-S shaped probability weighting function is depicted in Figure 1.1 (dashed curve). A neo-additive capacity, also depicted in Figure 1.1 (solid line), is a simple linear specification.

Under Choquet expected utility with a neo-additive capacity, an agent's preferences are represented by a weighted sum of the minimum utility, the maximum utility, and the expected utility. There is experimental evidence that preferences have this form. Lopes (1987) proposes a theory for risky choice that integrates two factors: a dispositional tendency to seek either security or potential and a situational aspiration level. Under Lopes' theory, the security/potential factor re-

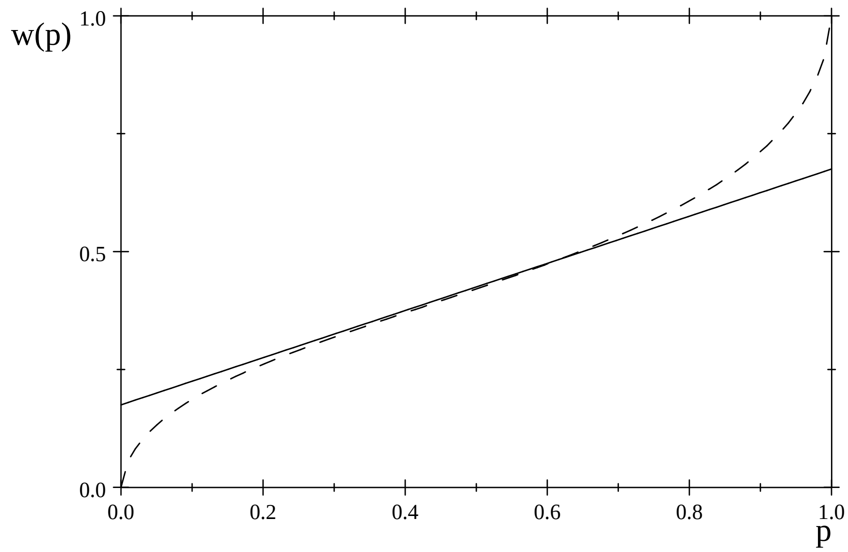

Figure 1.1: Neo-additive Capacity as Linear Probability Weighting Function

flects how a person weights the worst and best outcomes, while the aspiration level reflects an assessment of what outcome is reasonable to expect under the circumstances. On the basis of her theory, Lopes makes predictions about preferences over lotteries. She then presents experimental evidence consistent with her predictions.

Ambiguity, optimism, and pessimism. An advantage of representing beliefs with a neo-additive capacity is that it allows us to define concrete notions of ambiguity, optimism, and pessimism. Ambiguity corresponds to an agent's lack of confidence in his belief about the probability of uncertain events. Optimism and pessimism are defined according to the weights applied to extreme outcomes. Optimism means the weight applied to the best outcome exceeds the probability of the best outcome and pessimism means the weight applied to the worst outcome exceeds the probability of the worst outcome.[25]

[25] In general, the agent may overweight both the best and worst outcomes and underweight non-extreme outcomes. In the present model, however, in which there are only two outcomes, the injurer overweights either the accident outcome or the no accident outcome, but not both. A disadvantage of a neo-additive capacity is that it only allows the best and worst outcomes to be overweighted. A more general type of capacity called a *JP-capacity* allows a number of good

The usage of the terms "optimism" and "pessimism" in the present framework is consistent with their usage within an expected utility framework, in which optimism means the agent's subjective probability of a favorable (unfavorable) outcome is greater (less) than the objective probability of that outcome and pessimism means the agent's subjective probability of a favorable (unfavorable) outcome is less (greater) than the objective probability of that outcome (see, e.g., Posner 2003; Bar-Gill 2006). In both frameworks, optimism and pessimism correspond to "incorrect" decision weights resulting from misweighted or misperceived probabilities. The key distinction lies in their interpretation. In the present framework, optimism and pessimism properly may be interpreted as attitudes toward or reactions to ambiguity. Specifically, optimism corresponds to a concave (superadditive) capacity which reflects ambiguity loving while pessimism corresponds to a convex (subadditive) capacity which reflects ambiguity aversion (see Schmeidler 1989; Wakker 2001). However, as illustrated above, ambiguity attitudes cannot be captured within an expected utility framework because beliefs are represented with probabilities. In the expected utility framework, therefore, optimism and pessimism may not be interpreted as reactions to ambiguity.

Another advantage of a neo-additive capacity is that it parameterizes ambiguity, optimism, and pessimism. In the model, we interpret δ as the degree of ambiguity because it is the complement of the injurer's degree of confidence in π. We interpret α and $1-\alpha$ as the injurer's degrees of pessimism and optimism because they determine the respective weights assigned to the accident and no accident outcomes.

We can further motivate interpreting δ, α, and $1-\alpha$ as the injurer's degrees of ambiguity, pessimism, and optimism by reference to a *multiple priors* version of

and bad outcomes to be overweighted (see Eichberger and Kelsey 2009). However, when there are only two outcomes, as in the present model, a JP-capacity is isomorphic to a neo-additive capacity.

the model, in which the injurer's beliefs about accident risk are represented by a set of probabilities "centered" around π (see Chateauneuf et al. 2007; Eichberger and Kelsey 2009). It can be shown that equation (1.1) is equivalent to

$$V_\pi(c) = \alpha \min_{p \in \mathcal{P}(\pi)} E_p(c) + (1-\alpha) \max_{p \in \mathcal{P}(\pi)} E_p(c) \tag{1.3}$$

where $\mathcal{P}(\pi) = \{p \in [0,1] : (1-\delta)\pi \leq p \leq \delta + (1-\delta)\pi\}$ represents the set of accident probabilities p the injurer considers possible and $E_p(c)$ denotes the expected utility with respect to p of exercising level of care c given the applicable liability rule (see Eichberger and Kelsey 2009). In this version, we interpret δ as the degree of ambiguity because it determines and measures the size of the set $\mathcal{P}(\pi)$. If $\delta = 0$ the injurer unambiguously believes the probability of an accident is π (i.e., $\mathcal{P}(\pi) = \{\pi\}$). As $\delta \to 1$ the injurer considers an increasing range of accident probabilities to be possible. For $\delta = 1$ he believes all probabilities are possible (i.e., $\mathcal{P}(\pi) = [0,1]$). We interpret α and $1-\alpha$ as the injurer's degrees of pessimism and optimism because they correspond to the respective weights the injurer applies to the minimum and maximum expected utility over the set $\mathcal{P}(\pi)$. Stated another way, we interpret α and $1-\alpha$ as degrees of pessimism and optimism because they reflect the respective degrees to which the injurer evaluates the expected utility of exercising level of care c by the lowest and highest accident probabilities he considers possible.

Relationship to other models. Choquet expected utility with a neo-additive capacity μ based on a probability distribution p contains as special cases or is mathematically equivalent to several alternative models of decision making under uncertainty, including but not limited to: (i) subjective expected utility where p represents the agent's beliefs, if $\delta = 0$ and $\alpha \in [0,1]$; (ii) α-maxmin expected utility with multiple priors (Ghirardato et al. 2004) where the set of priors is $\mathcal{D} = \{q \in \triangle : q \geq (1-\delta)p\}$ (see Chateauneuf et al. 2007; Eichberger and Kelsey 2009); (iii) rank dependent expected utility with probability weighting function $\omega(p) = \mu(p)$ (see Wakker 1990); and (iv) cumulative prospect theory with probability weighting function $\omega(p) = \mu(p)$ and symmetric treatment of gains and losses (see Tversky and Wakker 1995).[26]

1.2 RESULTS

In order to establish the benchmark for comparison, I first derive the socially optimal level of care and state the results of the model in the absence of ambiguity, which correspond to the standard results of the basic unilateral accident model. I then derive the results of the model in the presence of ambiguity.

1.2.1 Socially Optimal Level of Care

In conformity with Shavell (1987) and others, I assume the social goal is to minimize total accident costs $c + L(c)$. That is, I assume the socially optimal level of care c^* solves

$$\min_{c \geq 0} \; c + L(c). \tag{1.4}$$

Assuming c^* is positive, it is implicitly defined by the first-order condition

$$-L'(c^*) = 1. \tag{1.5}$$

Condition (1.5) requires that the marginal reduction in expected accident losses—i.e., the marginal benefit of care—equals the marginal cost of care. Note that condition (1.5) defines the socially optimal level of care whether accident losses

[26] Additional relationships to other well-known models are identified in the Appendix.

are fixed or variable.[27]

1.2.2 Results Without Ambiguity

As noted above, in the absence of ambiguity ($\delta = 0$) the model reduces to the basic unilateral accident model. In both cases on accident losses, therefore, the results of the model without ambiguity correspond to the standard results of the basic unilateral accident model, which may be summarized as follows.

PROPOSITION 1.1 (SHAVELL 1987) *Under a rule of no liability, the injurer will exercise no care. Under strict liability, the injurer will exercise the socially optimal level of care. Under a negligence rule with the standard of due care set equal to the socially optimal level of care, the injurer will exercise the socially optimal level of care.*

PROOF. Under each liability rule, the injurer's problem is to choose the level of care that maximizes the expected outcome of engaging in his activity. Because the injurer is risk neutral and his payoff k is fixed, the injurer's problem is equivalent to minimizing his expected costs, which equal his cost of care plus his expected liability. Specifically, under a rule of no liability, the injurer's problem is

$$\min_{c \geq 0} \ c \tag{1.6}$$

and the injurer will choose $c^{NL} = 0 < c^*$. Under strict liability, the injurer's

[27] In contrast to the standard model, in the present model minimizing total accident costs is not necessarily equivalent to maximizing the sum of the utilities of the injurer and the victim. They do not coincide if the injurer faces ambiguity about accident risk and he is either optimistic or pessimistic (that is, if $\delta > 0$ and $\alpha \neq \pi$). However, c^* is the level of care that would be chosen by a rational social planner as part of a Pareto optimal allocation (see Appendix). Accordingly, I maintain that minimizing total accident costs is the appropriate social goal. Note that Posner (2003), Eide (2005), and Bigus (2006) take the same view. But compare Eide (2007).

problem is

$$\min_{c \geq 0} c + L(c). \tag{1.7}$$

Assuming the solution is positive, it is implicitly defined by the condition $-L'(c^{SL}) = 1$. Together with condition (1.5), this implies the injurer will choose $c^{SL} = c^*$ because $L''(c) > 0$. Under a negligence rule, the injurer's problem is

$$\min_{c \geq 0} \begin{cases} c & \text{if } c \geq \bar{c} \\ c + L(c) & \text{if } c < \bar{c} \end{cases} . \tag{1.8}$$

If the court sets the standard of due care equal to the socially optimal level of care ($\bar{c} = c^*$), then the injurer will choose $c^N = c^*$ because $c^* = \arg\min_{c \geq c^*} c$ and, given our assumptions, $c^* < c^* + L(c^*) \leq \min \{c + L(c) : c \in [0, c^*)\}$. ■

It is obvious why the injurer will exercise no care under a rule of no liability. Under strict liability, the injurer's marginal benefit of care equals the social marginal benefit of care. Consequently, strict liability induces the injurer to take optimal care. The reason the injurer takes optimal care under a negligence rule with the standard of due care set equal to the socially optimal level of care is twofold. First, the injurer will not exercise too much care because he faces no liability if his level of care is at or above the socially optimal level of care. Second, the injurer will not exercise too little care because he faces strictly liability if his level of care is below the socially optimal level of care, and strict liability induces him to exercise the socially optimal level of care.

The results of Proposition 1.1 are illustrated by Figure 1.2, which is a variation of a classic diagram from the tort law and economics literature (see, e.g., Shavell 1987; Miceli 1997). The curve $c + L(c)$ represents the injurer's expected cost

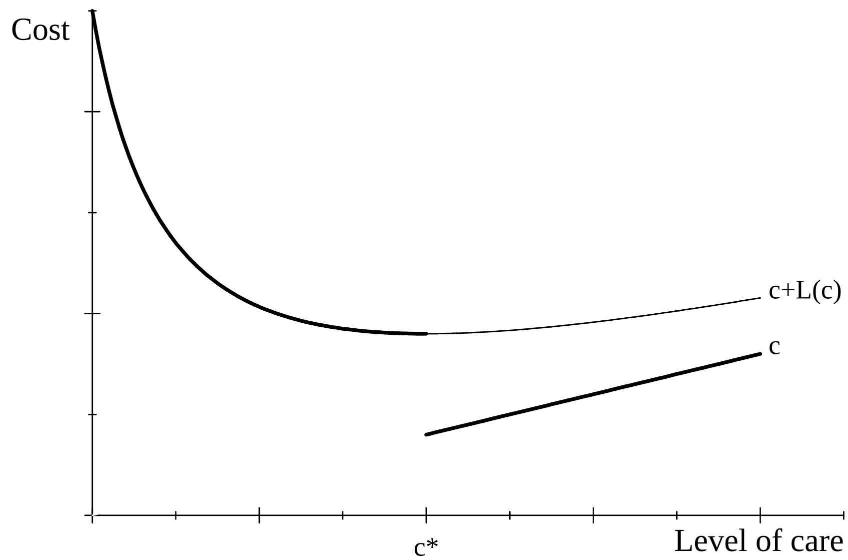

Figure 1.2: Efficient Care under Strict Liability and Negligence

schedule under strict liability, while the thick portion of $c + L(c)$ plus the line segment c represent the injurer's expected cost schedule under a negligence rule with the standard of due care set equal to the socially optimal level of care. As shown in Figure 1.2, under each liability rule the injurer's expected cost schedule attains its minimum at the socially optimal level of care, c^*.

1.2.3 Results under Ambiguity

I now consider the injurer's behavior under ambiguity ($\delta > 0$). I analyze separately the cases of fixed accident losses and variable accident losses. In the case of fixed accident losses, certain of the comparative statics results will depend on a property of $\pi(c)$ that I shall call *local convexity* and measure by $\rho_\pi(c) \equiv -\frac{\pi''(c)}{\pi'(c)}$. We may view $\rho_\pi(c)$ as a measure of the local convexity of $\pi(c)$ because, loosely speaking, it measures the degree of curvature of π at c.[28] I shall say that $\pi(c)$ exhibits increasing

[28]This view of $\rho_\pi(c)$ is motivated by the standard interpretation of the Arrow-Pratt coefficient of absolute risk aversion $r_u(x) = -\frac{u''(x)}{u'(x)}$ as a measure of the curvature of the utility function u at x (see, e.g., Mas-Colell et al. 1995, p. 190).

(decreasing) local convexity if $\rho_\pi(c)$ is an increasing (decreasing) function of c.

A. Fixed Accident Losses

No liability. Under a rule of no liability, whether or not an accident occurs, the outcome for the injurer is the same: $m(c) = M(c) = k - c$. It follows from equation (1.1) that the injurer's problem is $\max_{c \geq 0} k - c$, which is equivalent to problem (1.6). Hence, he will choose $c^{NL} = 0 < c^*$. That is, the injurer will exercise no care.

Strict liability. Under strict liability, the worst outcome is $m(c) = k - c - l$ and the best outcome is $M(c) = k - c$. Accordingly, by equation (1.1) the injurer's problem is

$$\max_{c \geq 0} \delta\alpha(k-c-l) + \delta(1-\alpha)(k-c) + (1-\delta)[\pi(c)(k-c-l) + (1-\pi(c))(k-c)], \quad (1.9)$$

which is equivalent to

$$\min_{c \geq 0} c + \delta\alpha l + (1-\delta)L(c). \quad (1.10)$$

Assuming $\delta < 1$ and the solution to (1.10) is positive, it is implicitly defined by the first-order condition

$$-L'(c^{SL}) = \frac{1}{1-\delta}. \quad (1.11)$$

Condition (1.11) implies the following results.

Proposition 1.2 *In the case of fixed accident losses, the level of care exercised by the injurer under strict liability will be less than the socially optimal level of care. In addition, the injurer's level of care decreases with the degree of ambiguity. As a result, the difference between the socially optimal level of care and the injurer's level of care increases with the degree of ambiguity. Furthermore, while the injurer's level of care increases with the magnitude of accident losses, so does the difference* *between the socially optimal level of care and the injurer's level of care if $\pi(c)$ exhibits decreasing local convexity.*

PROOF. By assumption, $\delta \in (0,1)$. Thus, conditions (1.5) and (1.11) imply $-L'(c^{SL}) = \frac{1}{1-\delta} > 1 = -L'(c^*)$, which in turn implies $c^{SL} < c^*$ because $L''(c) > 0$.

Implicitly differentiating conditions (1.5) and (1.11) with respect to δ, we have $\frac{\partial c^{SL}}{\partial \delta} = -\frac{1}{L''(c^{SL})(1-\delta)^2} < 0 = \frac{\partial c^*}{\partial \delta}$ because $L''(c) > 0$. Hence, c^{SL} is decreasing in δ and the difference $c^* - c^{SL}$ is increasing in δ.

Implicitly differentiating conditions (1.5) and (1.11) with respect to l, we have $\frac{\partial c^*}{\partial l} = -\frac{\pi'(c^*)}{\pi''(c^*)l} > 0$ and $\frac{\partial c^{SL}}{\partial l} = -\frac{\pi'(c^{SL})}{\pi''(c^{SL})l} > 0$ because $\pi'(c) < 0$ and $\pi''(c) > 0$. Furthermore, if $\rho_\pi(c) = -\frac{\pi''(c)}{\pi'(c)}$ is a decreasing function of c, then $\frac{\partial c^*}{\partial l} = \frac{1}{\rho_\pi(c^*)l} > \frac{1}{\rho_\pi(c^{sl})l} = \frac{\partial c^{SL}}{\partial l}$ because $c^{SL} < c^*$. Therefore, although c^{SL} is increasing in l, the difference $c^* - c^{SL}$ is increasing in l if $\pi(c)$ exhibits decreasing local convexity. ■

The intuition behind Proposition 1.2 is straightforward. If we rewrite condition (1.11) as $-(1-\delta)\pi'(c^{SL})l = 1$, we see that ambiguity leads the injurer to discount the marginal benefit of care. As a result, the injurer will exercise too little care. An increase in the degree of ambiguity increases the ambiguity discount but does not affect the marginal benefit of care. Consequently, the injurer reduces his level of care further below the unchanged socially optimal level of care. On the other hand, an increase in the magnitude of accident losses increases the marginal benefit of care but does not affect the ambiguity discount. Both the injurer's level of care and the socially optimal level of care increase in response to the increase in the marginal benefit of care. However, if the marginal benefit of care decreases too rapidly with the level of care, the ambiguity discount leads the injurer to increase his level of care by less than the increase in the socially optimal level of care.

Negligence. Under a negligence rule with the standard of due care set equal to the socially optimal level of care ($\bar{c} = c^*$), the injurer's problem is

$$\min_{c \geq 0} \begin{cases} c & \text{if } c \geq c^* \\ c + \delta\alpha l + (1-\delta)L(c) & \text{if } c < c^* \end{cases} . \tag{1.12}$$

Note that $c^* = \underset{c \geq c^*}{\arg\min}\, c$ and that $c^{SL} = \underset{c \in [0, c^*)}{\arg\min}\, c + \delta\alpha l + (1-\delta)L(c)$. It follows that the injurer will choose

$$c^N = \begin{cases} c^* & \text{if } c^* \leq F(c^{SL}) \\ c^{SL} < c^* & \text{if } c^* > F(c^{SL}) \end{cases} \tag{1.13}$$

where $F(c^{SL}) \equiv c^{SL} + \delta\alpha l + (1-\delta)L(c^{SL})$. Note that $F(c^{SL})$ is the injurer's expected costs if he is negligent. Therefore, it is the expected benefit of exercising due care. Condition (1.13) implies the following results.

PROPOSITION 1.3 *In the case of fixed accident losses, the injurer's level of care under a negligence rule with the standard of due care set equal to the socially optimal level of care will be less than or equal to the socially optimal level of care. The likelihood that the injurer will exercise too little care (i) increases with the degree of ambiguity if he is optimistic and decreases therewith if he is pessimistic, (ii) increases with his degree of optimism and decreases with his degree of pessimism, and (iii) increases with the magnitude of accident losses if* $F(c^{SL}) - c^{SL} < \frac{1}{\rho_\pi(c^*)}$ *and decreases therewith if the reverse inequality holds.*

PROOF. Recall that $F(c^{SL}) \equiv c^{SL} + \delta\alpha l + (1-\delta)L(c^{SL})$. Condition (1.13) immediately implies $c^N \leq c^*$ because model parameters exist such that $c^* \leq F(c^{SL})$ and $c^* > F(c^{SL})$. For example, suppose $\pi(c) = \frac{1}{1+c}$, $l = 49$, $\delta = \frac{24}{49}$, and $\alpha = \frac{1}{5}$. It follows that $c^* = 6.0 \leq 13.8 = F(c^{SL})$. Now suppose $\pi(c) = \frac{1}{1+c}$, $l = 49$, $\delta = \frac{45}{49}$, and $\alpha = \frac{1}{100}$. Then we have $c^* = 6.0 > 3.45 = F(c^{SL})$.

Now, by the envelope theorem, $\frac{\partial F(c^{SL})}{\partial \delta} = \alpha l - L(c^{SL}) = [\alpha - \pi(c^{SL})]l$. It follows that $\frac{\partial F(c^{SL})}{\partial \delta} < 0 = \frac{\partial c^*}{\partial \delta}$ if $\alpha < \pi(c^{SL})$ and that $\frac{\partial F(c^{SL})}{\partial \delta} > 0 = \frac{\partial c^*}{\partial \delta}$ if $\alpha > \pi(c^{SL})$. Thus, the likelihood that $F(c^{SL}) < c^*$, and therefore that $c^{SL} < c^*$, increases with δ if the injurer is optimistic and decreases with δ if the injurer is pessimistic.

In addition, by the envelope theorem, $\frac{\partial F(c^{SL})}{\partial \alpha} = \delta l > 0 = \frac{\partial c^*}{\partial \alpha}$. Hence, the likelihood that $F(c^{SL}) < c^*$, and therefore that $c^{SL} < c^*$, increases with $1 - \alpha$ and decreases with α.

Finally, by the envelope theorem, $\frac{\partial F(c^{SL})}{\partial l} = \delta\alpha + (1 - \delta)\pi(c^{SL})$, which implies $\frac{\partial F(c^{SL})}{\partial l} \gtreqless \frac{\partial c^*}{\partial l}$ as $\delta\alpha + (1 - \delta)\pi(c^{SL}) \gtreqless \frac{1}{\rho_\pi(c^*)l}$, or as $F(c^{SL}) - c^{SL} \gtreqless \frac{1}{\rho_\pi(c^*)}$. Thus, the likelihood that $F(c^{SL}) < c^*$, and therefore that $c^{SL} < c^*$, increases with l if $F(c^{SL}) - c^{SL} < \frac{1}{\rho_\pi(c^*)}$ and decreases with l if $F(c^{SL}) - c^{SL} > \frac{1}{\rho_\pi(c^*)}$. ■

The results of Proposition 1.3 may be understood as follows. With or without ambiguity, the injurer will never exercise too much care under a negligence rule with the standard of due care set equal to the socially optimal level of care because he can avoid liability simply by exercising due care. In the presence of ambiguity, however, the cost of exercising due care, c^*, may exceed the expected benefit, $F(c^{SL})$, in which case the injurer will exercise too little care. The likelihood that the injurer will exercise too little care increases with the difference $c^* - F(c^{SL})$. Because ambiguity affects the expected benefit, but not the cost, of exercising due care, variation in the likelihood that the injurer will exercise too little care in response to changes in ambiguity or the injurer's ambiguity attitude results from variation in the expected benefit of exercising due care. The expected benefit of exercising due care increases with ambiguity if the injurer is pessimistic because he reacts by further overweighting the accident outcome; it decreases with ambiguity if the injurer is optimistic because he reacts by further underweighting the accident outcome. Similarly, the expected benefit of exercising due care increases as the injurer becomes relatively more pessimistic because he reacts by increasing the weight on the accident outcome and decreases as he becomes relatively more optimistic because he reacts by decreasing the weight on the accident outcome. An increase in the magnitude of accident losses increases both the cost and the expected benefit of exercising due care. Whether the cost or the expected benefit increases at a faster rate depends on the relationship between $F(c^{SL}) - c^{SL}$, the injurer's expected liability if he is negligent, and $\frac{1}{\rho_\pi(c^*)}$, the inverse local convexity of $\pi(c)$ at c^*. When $F(c^{SL}) - c^{SL} < \frac{1}{\rho_\pi(c^*)}$ the cost of exercising due care increases more rapidly than the expected benefit, and when $F(c^{SL}) - c^{SL} > \frac{1}{\rho_\pi(c^*)}$ the expected benefit increases more rapidly than the cost. This is because $F(c^{SL}) - c^{SL}$ and $\frac{1}{\rho_\pi(c^*)}$ are in the same proportion as the marginal expected benefit and the marginal cost of exercising due care.[29]

B. Variable Accident Losses

No liability. The nature of accident losses is irrelevant under a rule of no liability. Thus, the injurer's problem with variable accident losses is identical to his problem with fixed accident losses, and the injurer will exercise no care—i.e., $c^{NL} = 0 < c^*$.

Strict liability. Under strict liability, the injurer's problem is

$$\max_{c \geq 0} c + \delta \alpha l(c) + (1 - \delta) L(c). \tag{1.14}$$

[29]To see this, note that $\frac{\partial F(c^{SL})}{\partial l} = \frac{F(c^{SL}) - c^{SL}}{l}$ and $\frac{\partial c^*}{\partial l} = \frac{1}{\rho_\pi(c^*) l}$.

Assuming $\delta < 1$ and the solution to (1.14) is positive, it is implicitly defined by the first-order condition

$$-L'(c^{SL}) = \frac{1 + \delta\alpha l'(c^{SL})}{1 - \delta}. \tag{1.15}$$

Assuming $\alpha > 0$, conditions (1.5) and (1.15) imply $-L'(c^{SL}) \gtreqqless -L'(c^*)$ as $-l'(c^{SL}) \lesseqqgtr \frac{1}{\alpha}$, which in turn implies

$$c^{SL} \lesseqqgtr c^* \text{ as } -l'(c^{SL}) \lesseqqgtr \frac{1}{\alpha} \tag{1.16}$$

because $L''(c) > 0$.[30] Conditions (1.15) and (1.16) imply the following results.

PROPOSITION 1.4 *In the case of variable accident losses, the injurer's level of care under strict liability may be less than, equal to, or greater than the socially optimal level of care, where equality is a borderline case. The injurer's level of care decreases with his degree of optimism and increases with his degree of pessimism. As a result, the likelihood that the injurer will exercise too little care increases with his degree of optimism and decreases with his degree of pessimism. Conversely, the likelihood that the injurer will exercise too much care decreases with his degree of optimism and increases with his degree of pessimism. The injurer's level of care decreases with the degree of ambiguity if he is optimistic or if he is pessimistic and* $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ *and increases therewith if he is pessimistic and* $\alpha > \frac{L'(c^{SL})}{l'(c^{SL})}$. *Accordingly, the likelihood that the injurer will exercise too little care increases with the degree of ambiguity if he is optimistic or if he is pessimistic and* $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ *and decreases therewith if he is pessimistic and* $\alpha > \frac{L'(c^{SL})}{l'(c^{SL})}$. *Conversely, the likelihood that the injurer will exercise too much care decreases with the degree of ambiguity if he is optimistic or if he is pessimistic and* $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ *and increases therewith if he is*

[30] Note that condition (1.15) requires $-l'(c^{SL}) < \frac{1}{\delta\alpha}$ because $L'(c) < 0$.

pessimistic and $\alpha > \frac{L'(c^{SL})}{l'(c^{SL})}$.

PROOF. Condition (1.16) immediately implies $c^{SL} \lesseqqgtr c^*$ because model parameters exist such that $-l'(c^{SL}) \lesseqqgtr \frac{1}{\alpha}$. For example, suppose $\pi(c) = \frac{1}{1+c}$, $l(c) = 1000e^{-c}$, $\delta = \frac{9}{10}$, and $\alpha = \frac{1}{9}$. It follows that $c^{SL} = 4.7896$ and $-l'(c^{SL}) = 1000e^{-4.7896} = 8.3 < 9 = \frac{1}{\alpha}$. Now suppose $\pi(c) = \frac{1}{1+c}$, $l(c) = 1000e^{-c}$, $\delta = \frac{9}{10}$, and $\alpha = \frac{2}{9}$. It follows that $c^{SL} = 5.385$ and $-l'(c^{SL}) = 1000e^{-5.385} = 4.6 > 4.5 = \frac{1}{\alpha}$. Finally, suppose $\pi(c) = \frac{1}{1+c}$, $l(c) = 1000e^{-c}$, $\delta = \frac{9}{10}$, and $\alpha = 0.18636$. It follows that $c^{SL} = c^* = 5.2277$ and $-l'(c^{SL}) = 1000e^{-5.2277} = 5.366 = \frac{1}{\alpha}$. Note that this is a borderline case because $(c, \alpha) = (5.2277, 0.18636)$ is the unique pair that simultaneously satisfies conditions (1.5) and (1.15).

Implicitly differentiating condition (1.15) with respect to α, we have $\frac{\partial c^{SL}}{\partial \alpha} = -\frac{\delta l'(c^{SL})}{(1-\delta)L''(c^{SL})+\delta\alpha l''(c^{SL})} > 0$ because $l'(c) < 0$, $l''(c) > 0$, and $L''(c) > 0$. Thus, c^{SL} is increasing in α and decreasing in $1-\alpha$. Furthermore, $\frac{\partial c^{SL}}{\partial \alpha} > 0$ and $l'(c) < 0$ imply $-l'(c^{SL})$ is increasing in α. Hence, since $\frac{1}{\alpha}$ is decreasing in α, the likelihood that $-l'(c^{SL}) < \frac{1}{\alpha}$, and therefore that $c^{SL} < c^*$, is increasing in $1-\alpha$ and decreasing in α. Conversely, the likelihood that $-l'(c^{SL}) > \frac{1}{\alpha}$, and therefore that $c^{SL} > c^*$, is decreasing in $1-\alpha$ and increasing in α.

Implicitly differentiating condition (1.15) with respect to δ, we have $\frac{\partial c^{SL}}{\partial \delta} = \frac{L'(c^{SL})-\alpha l'(c^{SL})}{(1-\delta)L''(c^{SL})+\delta\alpha l''(c^{SL})}$. Note that $(1-\delta)L''(c^{SL}) + \delta\alpha l''(c^{SL}) > 0$ because $L''(c) > 0$ and $l''(c) > 0$. So $\frac{\partial c^{SL}}{\partial \delta} \gtreqqless 0$ as $L'(c^{SL}) - \alpha l'(c^{SL}) \gtreqqless 0$, or as $\alpha \gtreqqless \frac{L'(c^{SL})}{l'(c^{SL})}$. Now $\frac{L'(c^{SL})}{l'(c^{SL})} = \frac{\pi'(c^{SL})l(c^{SL})+\pi(c^{SL})l'(c^{SL})}{l'(c^{SL})} = \frac{\pi'(c^{SL})l(c^{SL})}{l'(c^{SL})} + \pi(c^{SL}) > \pi(c^{SL})$ because $\pi'(c) > 0$ and $l'(c) > 0$. It follows that $\frac{\partial c^{SL}}{\partial \delta} < 0$ if $\alpha < \pi(c^{SL}) < \frac{L'(c^{SL})}{l'(c^{SL})}$ or if $\pi(c^{SL}) < \alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ and that $\frac{\partial c^{SL}}{\partial \delta} > 0$ if $\alpha > \frac{L'(c^{SL})}{l'(c^{SL})} > \pi(c^{SL})$. Thus, c^{SL} is decreasing in δ if the injurer is optimistic or if he is pessimistic and $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ and c^{SL} is increasing in δ if the injurer is pessimistic and $\alpha > \frac{L'(c^{SL})}{l'(c^{SL})}$.

Now $l'(c) < 0$ implies $-l'(c^{SL})$ is decreasing in δ if $\frac{\partial c^{SL}}{\partial \delta} < 0$ and increasing in δ if $\frac{\partial c^{SL}}{\partial \delta} > 0$. Thus, since $\frac{1}{\alpha}$ is independent of δ, it follows that the likelihood that $-l'(c^{SL}) < \frac{1}{\alpha}$, and therefore that $c^{SL} < c^*$, is increasing in δ if the injurer is optimistic or if he is pessimistic and $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ and decreasing in δ if the injurer is pessimistic and $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$. Conversely, the likelihood that $-l'(c^{SL}) > \frac{1}{\alpha}$, and therefore that $c^{SL} > c^*$, is decreasing in δ if the injurer is optimistic or if he is pessimistic and $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$ and increasing in δ if the injurer is pessimistic and $\alpha < \frac{L'(c^{SL})}{l'(c^{SL})}$. ■

To understand why ambiguity may lead the injurer to exercise too little or too much care under strict liability in the case of variable accident losses, rewrite condition (1.15) as $-(1-\delta)L'(c^{SL}) - \delta\alpha l'(c^{SL}) = 1$. We see that while ambiguity leads the injurer to discount the benefit from a marginal reduction in expected accident losses, it also leads the injurer to benefit from a marginal reduction in the magnitude of accident losses per se, which latter benefit increases with the injurer's degree of pessimism. If the latter benefit is sufficiently large and the injurer is sufficiently pessimistic, he will find it worthwhile to increase his level of care above the socially optimal level of care, notwithstanding the ambiguity discount on expected accident losses. Otherwise, as in the case of fixed accident losses, the ambiguity discount will cause the injurer to exercise too little care.

The comparative statics results on optimism, pessimism, and ambiguity are consistent with the corresponding results of Proposition 1.3 above and Proposition 1.5 below and form part of a general finding that the injurer's level of care decreases with his degree of optimism, increases with his degree of pessimism, and decreases or increases with the degree of ambiguity depending on whether he is optimistic or pessimistic, respectively.[31] This finding agrees with our basic intuition, for it seems

[31] It should be noted that, unlike in proposition 3 and proposition 5, the comparative statics result with respect to ambiguity in proposition 4 does not match the general finding precisely.

natural that if and to the extent the injurer reacts to ambiguity in an optimistic way by overweighing the no accident outcome he would tend to reduce his level of care, and if and to the extent the injurer reacts to ambiguity in a pessimistic way by overweighting the accident outcome he would tend to increase his level of care. It also agrees with the general thinking of legal scholars (see Posner 2003).

Negligence. Under a negligence rule with the standard of due care set equal to the socially optimal level of care, the injurer's problem is

$$\min_{c \geq 0} \begin{cases} c & \text{if } c \geq c^* \\ c + \delta\alpha l(c) + (1-\delta)L(c) & \text{if } c < c^* \end{cases} . \tag{1.17}$$

If $-l'(c^{SL}) \geq \frac{1}{\alpha}$, then the injurer will choose $c^N = c^* \leq c^{SL}$ because $c^* = \arg\min_{c \geq c^*} c$ and $c^* < c^* + \delta\alpha l(c^*) + (1-\delta)L(c^*) \leq \min\{c + \delta\alpha l(c) + (1-\delta)L(c) : c \in [0, c^*)\}$. If $-l'(c^{SL}) < \frac{1}{\alpha}$, then $c^{SL} < c^*$ and the injurer will choose

$$c^N = \begin{cases} c^* & \text{if } c^* \leq G(c^{SL}) \\ c^{SL} < c^* & \text{if } c^* > G(c^{SL}) \end{cases} , \tag{1.18}$$

where $G(c^{SL}) \equiv c^{SL} + \delta\alpha l(c^{SL}) + (1-\delta)L(c^{SL})$, because $c^* = \arg\min_{c \geq c^*} c$ and $c^{SL} = \arg\min_{c \in [0,c^*)} c + \delta\alpha l(c) + (1-\delta)L(c)$. The injurer's decision rules imply the following results.

Proposition 1.5 *In the case of variable accident losses, the injurer's level of care under a negligence rule with the standard of due care set equal to the socially optimal level of care will be less than or equal to the socially optimal level of care.*

However, it deviates only when $\pi(c^{SL}) < \alpha < \frac{L'(c^{SL})}{l'(c^{SL})} = \pi(c^{SL}) + \frac{\pi'(c^{SL})}{l'(c^{SL})} l(c^{SL})$, and this range will be narrow provided that the marginal reduction in the magnitude of accident losses is sufficiently greater than the marginal reduction in the probability of an accident at c^{SL} or that the magnitude of accident losses given c^{SL} is sufficiently small, either or both of which it seems reasonable to assume.

The likelihood that the injurer will exercise too little care (i) increases with his degree of optimism and decreases with his degree of pessimism and (ii) increases with the degree of ambiguity if he is optimistic and decreases therewith if he is pessimistic.

PROOF. Recall that $G(c^{SL}) \equiv c^{SL} + \delta\alpha l(c^{SL}) + (1-\delta)L(c^{SL})$. In the proof of Proposition 1.4, we show that model parameters exist such that $-l'(c^{SL}) \lesseqqgtr \frac{1}{\alpha}$. Thus, to establish that $c^N \leq c^*$, it is sufficient to show that model parameters exist such that $c^* > G(c^{SL})$. Suppose $\pi(c) = \frac{1}{1+c}$, $l(c) = 1000e^{-c}$, $\delta = \frac{99}{100}$, and $\alpha = \frac{1}{99}$. It follows that $c^* = 5.2277$ and $c^{SL} = 2.6058$,[32] and that $G(c^{SL}) = 2.6058 + \left(\frac{99}{100}\right)\left(\frac{1}{99}\right)(1000e^{-2.6058}) + \left(1 - \frac{99}{100}\right)\left(\frac{1}{1+2.6058}\right)(1000e^{-2.6058}) = 3.549 < 5.2277 = c^*$.

Now, by the envelope theorem, $\frac{\partial G(c^{SL})}{\partial \delta} = \alpha l(c^{SL}) - L(c^{SL}) = [\alpha - \pi(c^{SL})]l(c^{SL})$. It follows that $\frac{\partial G(c^{SL})}{\partial \delta} < 0 = \frac{\partial c^*}{\partial \delta}$ if $\alpha < \pi(c^{SL})$ and that $\frac{\partial G(c^{SL})}{\partial \delta} > 0 = \frac{\partial c^*}{\partial \delta}$ if $\alpha > \pi(c^{SL})$. Thus, the likelihood that $G(c^{SL}) < c^*$, and therefore that $c^{SL} < c^*$, increases with δ if the injurer is optimistic and decreases with δ if the injurer is pessimistic.

Finally, by Proposition 1.4, the likelihood that $-l'(c^{SL}) \geq \frac{1}{\alpha}$, and therefore that $c^N = c^*$, increases with α and decreases with $1-\alpha$. If $-l'(c^{SL}) < \frac{1}{\alpha}$, the likelihood that $c^* \leq G(c^{SL})$, and therefore that $c^N = c^*$, increases with α and decreases with $1-\alpha$ because, by the envelope theorem, $\frac{\partial G}{\partial \alpha} = \delta l(c^{SL}) > 0$. It follows that the likelihood that the injurer will choose $c^N = c^*$ increases with α and decreases with $1-\alpha$, which implies the likelihood that the injurer will choose $c^N < c^*$ increases with $1-\alpha$ and decreases with α. ■

The results of Proposition 1.5 follow from previous results. The injurer may exercise too little care under negligence for the same reason he may exercise too little care under strict liability: he discounts the marginal benefit of care. The in-

[32] Note that $-l'(c^{SL}) = 1000e^{-2.6058} = 73.844 < 99 = \frac{1}{\alpha}$.

jurer will never exercise too much care under negligence because, as noted above, he can avoid liability by exercising the socially optimal level of care. The comparative statics results on optimism, pessimism, and ambiguity support the finding that, in general, the injurer's level of care decreases with optimism, increases with pessimism, and decreases or increases with ambiguity depending on whether he is optimistic or pessimistic, respectively.

1.3 NUMERICAL EXAMPLE

This section develops a simple numerical example in order to illustrate the results of the model. Throughout the example, I assume:

- the probability of an accident is $\pi(c) = \frac{1}{1+c}$;
- the degree of ambiguity may be zero, low, or high: $\delta \in \{0, \frac{9}{25}, \frac{21}{25}\}$; and
- the injurer's degree of pessimism may be low or high: $\alpha \in \{\frac{1}{50}, \frac{39}{50}\}$.

Note that $\pi(c)$ is strictly decreasing, strictly convex, and exhibits decreasing local convexity.[33] In addition, it turns out that $\pi(c) \in \left(\frac{1}{50}, \frac{39}{50}\right)$ for each level of care the injurer may exercise. Accordingly, the injurer is optimistic when his degree of pessimism is low and pessimistic when his degree of pessimism is high.

1.3.1 Fixed Accident Losses

To begin, suppose accident losses are fixed and their magnitude may be low or high: $l \in \{25, 36\}$. Given our assumptions, the socially optimal level of care is $c^* = \sqrt{l} - 1$. Thus, $c^* = 4$ when accident losses are low and $c^* = 5$ when accident losses are high.

Table 1.1: Levels of Care under Strict Liability with Fixed Accident Losses

(l, δ)	c^*	c^{SL}
$(25, 0)$	4	4
$(25, \frac{9}{25})$	4	3
$(25, \frac{21}{25})$	4	1
$(36, 0)$	5	5
$(36, \frac{9}{25})$	5	3.8
$(36, \frac{21}{25})$	5	1.4

A. Strict Liability

Under strict liability, the injurer's level of care is $c^{SL} = \sqrt{(1-\delta)\,l} - 1$. Note that the injurer's level of care does not depend on his degree of pessimism, α. Table 1.1 sets forth the socially optimal level of care, c^*, and the injurer's level of care, c^{SL}, for each possible pair (l, δ). From Table 1.1 we can see that:

1. The injurer will take optimal care in the absence of ambiguity, but in the presence of ambiguity he will take too little care.

2. Increasing ambiguity progressively reduces the injurer's level of care below the socially optimal level of care. In particular, as the degree of ambiguity increases from zero to low to high, the injurer's level of care falls from 4 to 3 to 1 if accident losses are low and from 5 to 3.8 to 1.4 if accident losses are high.

3. While the injurer's level of care increases in response to an increase in the magnitude of accident losses, the gap between the injurer's level of care and the socially optimal level of care increases as well. Specifically, if accident losses increase from low to high, the socially optimal level of care increases by 1 while the injurer's level of care increases only by 0.8 if ambiguity is low and by 0.4 if ambiguity is high.

[33] To see this, note that $\pi'(c) = -\frac{1}{(1+c)^2} < 0$, $\pi''(c) = \frac{2}{(1+c)^3} > 0$, and $\rho'_\pi(c) = -\frac{2}{(1+c)^2} < 0$.

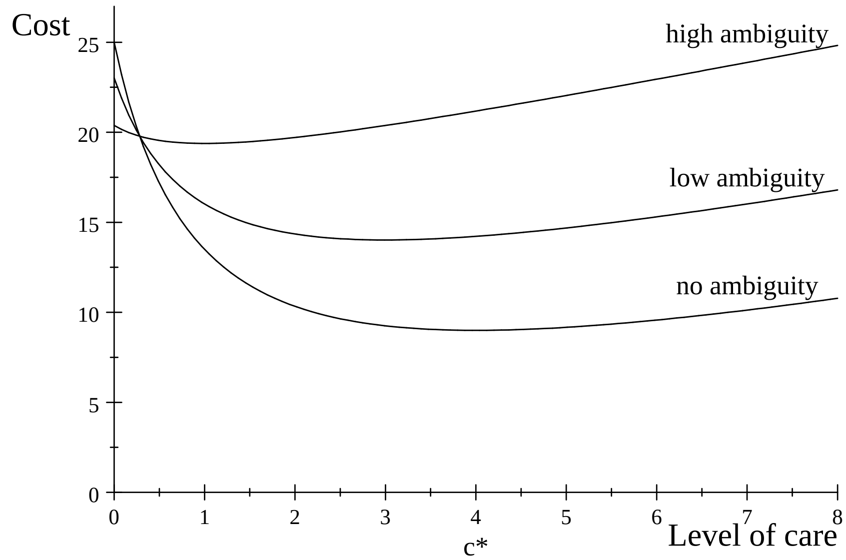

Figure 1.3: Effect on Care of Increasing Ambiguity under Strict Liability with Fixed Accident Losses

Each of these results is consistent with Proposition 1.2. Figure 1.3 illustrates the second result assuming low accident costs and a high degree of pessimism. Figure 1.4 illustrates the third result assuming a low degree of ambiguity and a high degree of pessimism.

B. Negligence

Under a negligence rule with the standard of due care set equal to the socially optimal level of care, the injurer will exercise due care if the cost, c^*, does not exceed the expected benefit, $F(c^{SL}) \equiv c^{SL} + \delta\alpha l + (1-\delta)L(c^{SL})$; otherwise he will exercise c^{SL}. Table 1.2 sets forth the relevant cost-benefit calculations and specifies the injurer's level of care, c^N, for each possible triple (l, δ, α). From Table 1.2 we can see that:

1. The injurer will take optimal care in the absence of ambiguity, if ambiguity is low, or if he is pessimistic. The injurer will exercise too little care only

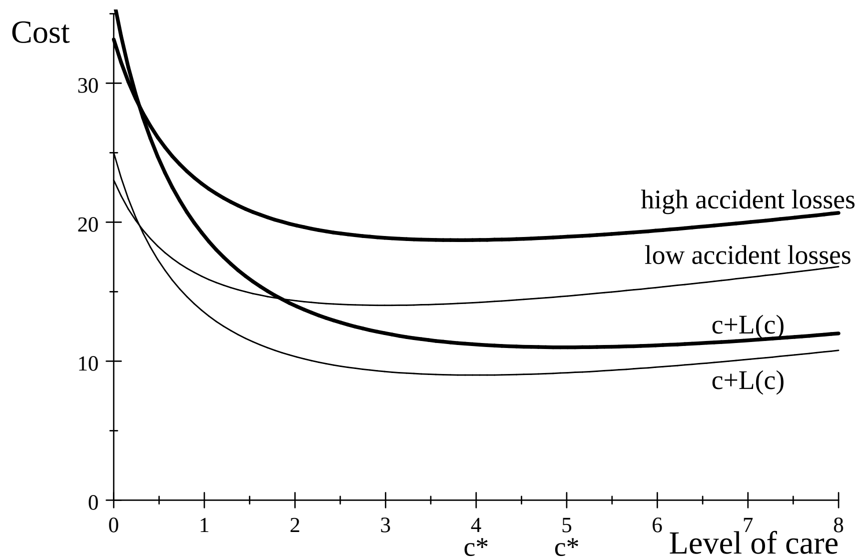

Figure 1.4: Effect on Care of Increasing Fixed Accident Losses under Strict Liability

Table 1.2: The Injurer's Level of Care under Negligence with Fixed Accident Losses

(l, δ, α)	c^*	c^{SL}	$F(c^{SL})$	c^N
$(25, 0, \frac{1}{50})$	4	4	9	4
$(25, 0, \frac{39}{50})$	4	4	9	4
$(25, \frac{9}{25}, \frac{1}{50})$	4	3	7.18	4
$(25, \frac{9}{25}, \frac{39}{50})$	4	3	14.02	4
$(25, \frac{21}{25}, \frac{1}{50})$	4	1	3.42	1
$(25, \frac{21}{25}, \frac{39}{50})$	4	1	19.38	4
$(36, 0, \frac{1}{50})$	5	5	11	5
$(36, 0, \frac{39}{50})$	5	5	11	5
$(36, \frac{9}{25}, \frac{1}{50})$	5	3.8	8.86	5
$(36, \frac{9}{25}, \frac{39}{50})$	5	3.8	18.71	5
$(36, \frac{21}{25}, \frac{1}{50})$	5	1.4	4.40	1.4
$(36, \frac{21}{25}, \frac{39}{50})$	5	1.4	27.39	5

when ambiguity is high and he is optimistic.

2. Increasing ambiguity progressively reduces the expected benefit of exercising due care if the injurer is optimistic and progressively increases it if he is pessimistic. In particular, if the injurer is optimistic, as the degree of ambiguity increases from zero to low to high the expected benefit of exercising due care falls from 9 to 7.18 to 3.42 if accident losses are low and from 11 to 8.86 to 4.40 if accident losses are high. If the injurer is pessimistic, however, as the degree of ambiguity increases from zero to low to high the expected benefit of exercising due care rises from 9 to 14.02 to 19.38 if accident losses are low and from 11 to 18.71 to 27.39 if accident losses are high.

3. In the presence of ambiguity, the expected benefit of exercising due care increases with the injurer's degree of pessimism. Specifically, when ambiguity is low, as the injurer's degree of pessimism increases from low to high the expected benefit of exercising due care rises from 7.18 to 14.02 if accident losses are low and from 8.86 to 18.71 if accident losses are high. When ambiguity is high, as the injurer's degree of pessimism increases from low to high the expected benefit of exercising due care leaps from 3.42 to 19.38 if accident losses are low and from 4.40 to 27.39 if accident losses are high.

4. While both the cost and the expected benefit of exercising due care increase in response to an increase in the magnitude of accident losses, the gap between them increases as well.

All four results are consistent with Proposition 1.3. Note that the second and third results are consistent because the likelihood that the injurer will exercise too little care varies inversely with the expected benefit of exercising due care in response to changes in ambiguity or the injurer's ambiguity attitude. To see that

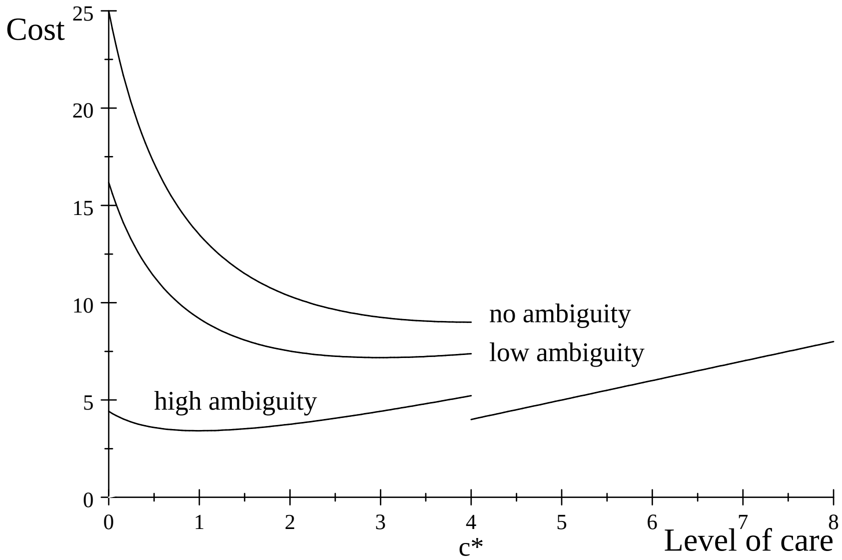

Figure 1.5: Effect on Care of Increasing Ambiguity under Negligence with Fixed Accident Losses

the fourth result is consistent, note that (i) when ambiguity is high and the injurer is optimistic, $\frac{1}{\rho_\pi(c^*=4)} = 2.5 > 2.42 = F(c^{SL}) - c^{SL}$ and the expected benefit of exercising due care increases by less than the cost when the magnitude of accident losses increases from low to high, and (ii) in all other cases, $F(c^{SL}) - c^{SL} > 2.5 = \frac{1}{\rho_\pi(c^*=4)}$ and the expected benefit of exercising due care increases by more than the cost when the magnitude of accident losses increases from low to high. Figure 1.5 illustrates how an optimistic injurer will take optimal care in the absence of ambiguity or if ambiguity is low, but will exercise too little care when ambiguity is high. It assumes accident losses are low.

1.3.2 Variable Accident Losses

Next suppose accident losses are variable and their magnitude is given by $l(c) = \frac{108}{1+c}$. Under this assumption, the socially optimal level of care is $c^* = 5$.

Table 1.3: Levels of Care under Strict Liability with Variable Accident Losses

(δ, α)	c^*	c^{SL}
$(0, \frac{1}{50})$	5	5
$(0, \frac{39}{50})$	5	5
$(\frac{9}{25}, \frac{1}{50})$	5	4.22
$(\frac{9}{25}, \frac{39}{50})$	5	6.06
$(\frac{21}{25}, \frac{1}{50})$	5	2.44
$(\frac{21}{25}, \frac{39}{50})$	5	7.65

A. Strict Liability

Table 1.3 sets forth the socially optimal level of care, c^*, and the injurer's level of care, c^{SL}, for each possible pair (δ, α). From Table 1.3 we can see that:

1. The injurer will take optimal care in the absence of ambiguity, but in the presence of ambiguity he will exercise too little care when he is optimistic and too much care when he is pessimistic.

2. For any given degree of ambiguity, the injurer's level of care increases with his degree of pessimism.

3. The injurer's level of care decreases with ambiguity when he is optimistic and increases with ambiguity when he is pessimistic.

Figures 1.6 and 1.7 collectively illustrate all three results, each of which is consistent with Proposition 1.4. In particular, with respect to the third result note that because $c^{SL} \geq 2.44$ we have $\alpha = \frac{39}{50} > \frac{2}{1+c^{SL}} = \frac{L'(c^{SL})}{l'(c^{SL})}$.

B. Negligence

Table 1.4 sets forth the relevant cost-benefit calculations and specifies the injurer's level of care, c^N, for each possible pair (δ, α). Recall that the injurer will

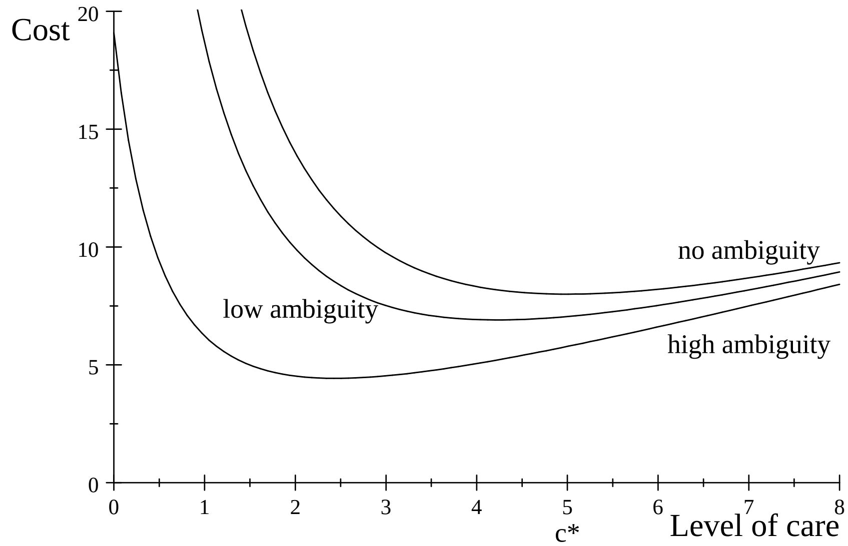

Figure 1.6: Effect on Care of Increasing Ambiguity under Strict Liability with Variable Accident Losses if the Injurer is Optimistic

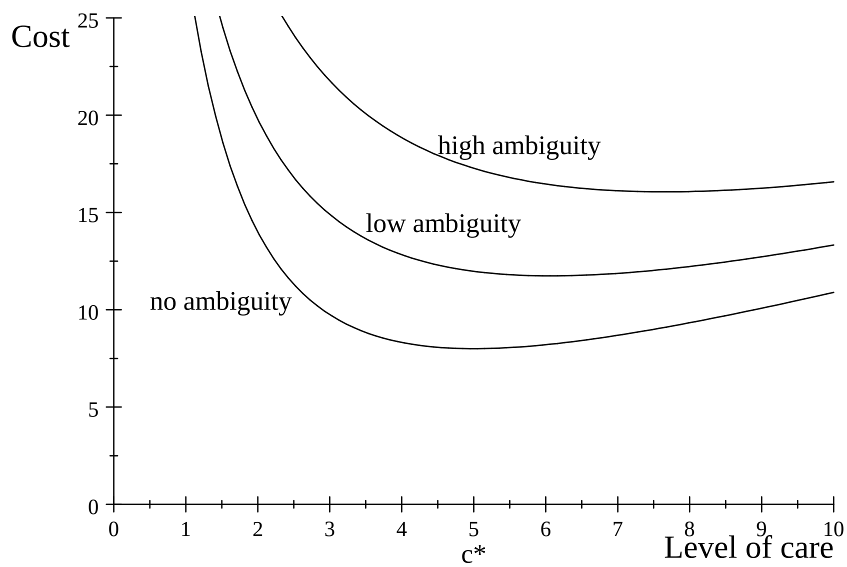

Figure 1.7: Effect on Care of Increasing Ambiguity under Strict Liability with Variable Accident Losses if the Injurer is Pessimistic

Table 1.4: The Injurer's Level of Care under Negligence with Variable Accident Losses

(δ, α)	c^*	c^{SL}	$G(c^{SL})$	c^N
$(0, \frac{1}{50})$	5	5	8	5
$(0, \frac{39}{50})$	5	5	8	5
$(\frac{9}{25}, \frac{1}{50})$	5	4.22	6.91	5
$(\frac{9}{25}, \frac{39}{50})$	5	6.06	11.74	5
$(\frac{21}{25}, \frac{1}{50})$	5	2.44	4.43	2.44
$(\frac{21}{25}, \frac{39}{50})$	5	7.65	16.06	5

exercise due care if $c^* \leq G(c^{SL}) \equiv c^{SL} + \delta\alpha l(c^{SL}) + (1-\delta)L(c^{SL})$; otherwise he will exercise c^{SL}. From Table 1.4 we can see that:

1. The injurer will take optimal care in the absence of ambiguity, if ambiguity is low, or if he is pessimistic. The injurer will exercise too little care only when ambiguity is high and he is optimistic.

2. In the presence of ambiguity, the expected benefit of exercising due care increases with the injurer's degree of pessimism. Specifically, when ambiguity is low, as the injurer's degree of pessimism increases from low to high the expected benefit of exercising due care rises from 6.91 to 11.74. When ambiguity is high, as the injurer's degree of pessimism increases from low to high the expected benefit of exercising due care leaps from 4.43 to 16.06.

3. Increasing ambiguity progressively reduces the expected benefit of exercising due care if the injurer is optimistic and progressively increases it if he is pessimistic. In particular, if the injurer is optimistic, as the degree of ambiguity increases from zero to low to high the expected benefit of exercising due care falls from 8 to 6.91 to 4.43. If the injurer is pessimistic, however, as the degree of ambiguity increases from zero to low to high the expected benefit of exercising due care rises from 8 to 11.74 to 16.06.

Note that each of the results is consistent with Proposition 1.5.

1.4 DISCUSSION

The results of the model suggest that neither strict liability nor negligence is generally efficient in the presence of ambiguity. Under both liability rules the injurer may exercise too little care, and under strict liability he even may exercise too much care in the case of variable accident losses. In addition, the injurer's level of care generally decreases with optimism, increases with pessimism, and decreases or increases with ambiguity depending on whether he is optimistic or pessimistic, respectively.

A further implication of the results is that negligence is more robust to ambiguity, which implies that negligence may be superior to strict liability in unilateral accident cases. Generally speaking, in the presence of ambiguity strict liability is never efficient (save only a borderline case when accident losses are variable), while negligence is efficient for a range of model parameter values. More specifically, the results suggest that although we might expect the two liability rules to perform equally poorly to the extent that optimism is the prevailing attitude toward ambiguity with respect to accident risk (as the psychology literature appears to suggest), we would expect negligence to outperform strict liability in unilateral accident contexts in which people are sufficiently pessimistic (for example, where an accident is highly available in the sense used in the psychology literature). Of course, an important countervailing factor is that strict liability is less costly for a court to implement. Nevertheless, the implication that negligence is more robust to ambiguity may help explain why it is the predominant liability rule in modern tort law.

Whatever the relative merits of strict liability and negligence, the basic result remains that neither liability rule is generally efficient when the injurer faces ambiguity with respect to accident risk. The exercise, then, is to design a liability rule that will induce the injurer to exercise optimal care in the face of ambiguity. One approach is to modify strict liability or negligence to include an adjustment to damages, which may be positive or negative, that equates the injurer's costs with total accident costs. Let h denote such adjustment. In the case of variable accident losses,[34] h is implicitly defined by

$$c + \delta\alpha(l(c) + h) + (1 - \delta)[\pi(c)(l(c) + h)] = c + \pi(c)l(c). \tag{1.19}$$

Rearranging terms, we have

$$h = \frac{\delta l(c)[\pi(c) - \alpha]}{\delta\alpha + (1 - \delta)\pi(c)}. \tag{1.20}$$

It follows immediately from equation (1.20) that

$$h \gtreqless 0 \text{ as } \pi(c) \gtreqless \alpha \tag{1.21}$$

and that

$$|h| \text{ increases with } |\pi(c) - \alpha|\,. \tag{1.22}$$

That is, (i) the adjustment will be positive when the injurer is optimistic, negative when he is pessimistic, and zero when he is neither optimistic nor pessimistic and (ii) the absolute magnitude of the adjustment increases as the injurer becomes more optimistic or pessimistic, as the case may be. Consequently, in light of the comparative statics results on optimism and pessimism, we may interpret h as a scaled (by the magnitude of accident losses) *ambiguity adjustment* that operates to punish optimism and the resulting tendency to decrease care and to reward pessimism

[34] I derive h for the case of variable accident losses because it is the more general case. The case of fixed accident losses is the special case where $l(c) = l$ for some scalar $l > 0$.

and the resulting tendency to increase care. It is straightforward to demonstrate the efficiency of an ambiguity adjusted rule of strict liability or negligence. Under an ambiguity adjusted rule of strict liability, the injurer faces problem (1.7) above and accordingly will choose $c^{SL} = c^*$. Under an ambiguity adjusted negligence rule, the injurer faces problem (1.8) above and accordingly will choose $c^N = c^*$.

It is worth noting that modifying negligence to include an adjustment to the standard of due care rather than to damages is not a viable alternative approach. Suppose ambiguity would lead an injurer to exercise suboptimal care under a negligence rule with the standard of due care set equal to the socially optimal level of care. A downward adjustment of the standard of due care below the socially optimal level of care could not induce the injurer to exercise the socially optimal level of care because the injurer's level of care would never exceed the standard of due care under a rule of negligence. To see that an upward adjustment of the standard of due care above the socially optimal level of care could not induce the injurer to exercise the socially optimal level of care, consider the case of variable accident losses (which includes fixed accident losses as a special case). By assumption, the injurer would choose $c^N < c^*$ under a negligence rule with $\bar{c} = c^*$. This implies that $c^N = c^{SL} < c^*$, that the injurer's decision rule is given by $c^N = \begin{cases} \bar{c} & \text{if } \bar{c} \leq G(c^{SL}) \\ c^{SL} < \bar{c} & \text{if } \bar{c} > G(c^{SL}) \end{cases}$, and that $\bar{c} > G(c^{SL})$. It follows that increasing $\bar{c}$ such that $\bar{c} > c^*$ would not induce the injurer to choose $c^N = c^*$ because it still would be the case that $\bar{c} > G(c^{SL})$ and, therefore, the injurer still would choose $c^N = c^{SL} < c^*$.

We can imagine (at least) two objections to an ambiguity adjusted liability rule. First, one could object that it is unworkable because the adjustment requires the court to determine the injurer's degree of and attitude toward ambiguity—δ and α—which are unobservable. While the unobservability of δ and α may preclude the court from perfectly implementing the ambiguity adjustment, the court conceivably could use observable characteristics of the injurer as proxies or instruments for δ and α in a "second-best" implementation.[35] Second, one could object that an ambiguity adjusted liability rule is unfair because the adjustment to damages is based not on the injurer's acts (i.e., his level of care) but rather on his beliefs (i.e., his degree of and attitude toward ambiguity). Many legal determinations, however, are based on a person's state of mind. In criminal cases, for example, whether a harmful act constitutes a crime, and quite often the degree of criminal liability, depends on the defendant's state of mind, or mens rea. Perhaps more on point, courts may award exemplary or punitive damages in tort cases on the basis that the harmful act was intentional, willful, wanton, or malicious. In addition, the purpose of the ambiguity adjustment is not to punish or reward the injurer's beliefs, but to cause the injurer to internalize the external social costs of suboptimal care. In this sense, it is analogous to a Pigouvian corrective tax and subsidy scheme and may be justified on the same grounds.

1.5 CONCLUDING REMARKS

This chapter generalizes the basic unilateral accident model of tort law and economics to allow for ambiguity with respect to accident risk. Standard formulations of the basic accident model are based on the expected utility framework, in which agents' beliefs about the likelihood of uncertain events are represented by probabilities. As a result, the standard models do not allow for ambiguity with respect to accident risk and cannot accommodate optimistic or pessimistic attitudes toward ambiguity. The Ellsberg paradox and related experimental evidence, however,

[35] In addition, recall that, in conformity with the basic accident model, I assume the court can accurately determine all relevant facts, including the agents' preferences. Under this assumption, which is central to the basic accident model, the court would be able to determine δ and α and perfectly implement the ambiguity adjustment.

suggest the importance of ambiguity attitudes for individual decision making generally. Moreover, psychology research suggests that people exhibit optimism and pessimism in the accident context.

To incorporate ambiguity into the basic unilateral accident model, I recast the model in the Choquet expected utility framework and represent the injurer's beliefs about accident risk with a neo-additive capacity. Choquet expected utility is a generalization of expected utility that allows for ambiguity. Under Choquet expected utility theory, agents' beliefs about the likelihood of uncertain events are represented by a non-additive probability, or a capacity. The non-additivity of the capacity allows for different ambiguity attitudes. A neo-additive capacity is a special type of capacity that is based on a probability distribution. That is, it is a probability weighting function. Choquet expected utility with a neo-additive capacity amounts to a weighted sum of the minimum utility, the maximum utility, and the expected utility with respect to the probability distribution on which the neo-additive capacity is based.

I represent the injurer's beliefs with a neo-additive capacity for several reasons. First, numerous empirical studies indicate that individuals weight probabilities in a manner consistent with a neo-additive capacity, and there is experimental evidence that preferences have the form suggested by Choquet expected utility with a neo-additive capacity. Second, a neo-additive capacity lends itself to concrete notions of ambiguity, optimism, and pessimism. Ambiguity corresponds to an agent's lack of confidence in his belief about the probability of uncertain events, while optimism and pessimism correspond to an agent overweighting the best and worst outcomes, respectively. Moreover, a neo-additive capacity parameterizes ambiguity, optimism, and pessimism, allowing us to perform comparative statics on changes in their degrees. Finally, Choquet expected utility with a neo-additive capacity is tractable and also quite general in that it includes as special cases or is mathematically equivalent to a number of alternative models of decision making under uncertainty.

The central result of the model is that, in the basic unilateral accident setting, neither strict liability nor negligence is generally efficient in the presence of ambiguity. This is in contrast with the standard results of the basic unilateral accident model, namely that both strict liability and negligence are efficient. In particular, I show that (i) under strict liability, the injurer will exercise too little care in the case of fixed accident losses and may exercise too little or too much care in the case of variable accident losses and (ii) under a negligence rule with the standard of due care set equal to the socially optimal level of care, the injurer may exercise too little care in both cases on accident losses. In addition, I find that, in general, the injurer's level of care decreases with his degree of optimism, increases with his degree of pessimism, and decreases or increases with the degree of ambiguity depending on whether the injurer is optimistic or pessimistic, respectively.

The basic intuition behind the main results may be summarized as follows. Ambiguity has two effects on the injurer's incentives to take care under strict liability. On the one hand, ambiguity leads the injurer to discount the benefit from a reduction in expected accident losses, which causes the injurer to tend to reduce his level of care. On the other hand, ambiguity leads the injurer to benefit from a reduction in the magnitude of accident losses per se, which causes the injurer to tend to increase his level of care. In the case of fixed accident losses, the injurer cannot affect the magnitude of accident losses, so only the former effect applies and the injurer will exercise too little care. In the case of variable accident losses, whether the injurer exercises too little or too much care depends on which of the two effects dominates. The former effect will dominate and the injurer will exercise too little care if the marginal benefit from a reduction in expected accident losses exceeds the marginal benefit from a reduction in the magnitude of accident losses and the injurer is optimistic or not too pessimistic. Otherwise, the latter effect will dominate and the injurer will exercise too much care. The injurer will take optimal care only in the borderline case where the two effects perfectly offset. Under a negligence rule with the standard of due care set equal to the socially optimal level of care, the injurer faces no liability if he satisfies the standard of due care and faces strict liability otherwise. Accordingly, if per the above analysis the injurer would exercise optimal care or even too much care under strict liability, then he will take optimal care under negligence. If, however, the injurer would exercise too little care under strict liability, he may or may not exercise optimal care under negligence depending on whether the expected benefit of exercising due care, which equals his expected costs of being negligent, exceeds the cost of exercising due care, which is the price of facing no liability.

A key implication of the results of the model is that negligence is more robust to ambiguity. This suggests that negligence may be superior to strict liability in unilateral accident cases. It also may help explain why negligence is the general basis for accident liability under modern Anglo-American tort law.

The model's results and implications aside, a principal contribution of this chapter is that it proposes a method to generalize the basic accident model to allow for ambiguity with respect to accident risk. The scope of the model presented in this chapter is limited to the case of unilateral accidents with fixed activity levels. Natural extensions of this chapter, therefore, include introducing ambiguity in the case of unilateral accidents with variable activity levels and in the more general case of bilateral accidents, including bilateral care and harm, with fixed and variable activity levels. In addition, future research could examine the implications of ambiguity for the economic analysis of other basic areas of law such as contracts, property, and criminal law, as well as other traditional law and economics topics such as litigation and settlement.

APPENDIX

A.1 CHOQUET EXPECTED UTILITY FRAMEWORK

A.1.1 General Framework

Let S be a non-empty, finite set of states. Associated with S is a set of events $\mathcal{E}$, which we take to be the power set of S. Let $X \subset \mathbb{R}$ be a non-empty, finite set of outcomes and let $\mathcal{F} = \{f : S \to X\}$ be a set of simple functions from states to outcomes, called simple acts. Let $u : X \to \mathbb{R}$ be a monotone increasing function from outcomes to real numbers. We interpret u as a Bernoulli utility function.

We now define a capacity and the Choquet integral of a simple act with respect to a capacity.

Definition A.1 (Capacity) *A* capacity *is a function $\mu : \mathcal{E} \to \mathbb{R}$ that satisfies:*

(i) monotonicity: $E, F \in \mathcal{E}$ and $E \subseteq F$ imply $\mu(E) \leq \mu(F)$; and

(ii) normalization: $\mu(\varnothing) = 0$ and $\mu(S) = 1$.

Note that a probability distribution is a special case of a capacity that satisfies additivity: $E, F \in \mathcal{E}$ and $E \cap F = \varnothing$ imply $\mu(E) + \mu(F) = \mu(E \cup F)$.

Definition A.2 (Choquet integral) *Let $f : S \to X$ be a simple act that takes on the values $x_1 \geq \cdots \geq x_n$. The* Choquet integral *of f with respect to a capacity μ is defined as*

$$\int f d\mu := \sum_{i=1}^{n} x_i[\mu(\{s \in S | f(s) \geq x_i\}) - \mu(\{s \in S | f(s) > x_i\})].$$

We interpret the Choquet integral as the expected value of the simple act f with respect to the capacity μ. The Choquet integral of the composition $u(f(s))$ with respect to the capacity μ is defined as the Choquet expected utility of f with respect to μ.[36]

A neo-additive capacity is a special type of capacity that is based on a probability distribution.

Definition A.3 (Neo-additive capacity) *Let δ, α be real numbers such that $0 \leq \delta, \alpha \leq 1$. A* neo-additive capacity ν *based on a probability distribution p is defined as*

$$\nu(E) := \begin{cases} 0 & \text{for } E = \varnothing \\ \delta(1-\alpha) + (1-\delta)p(E) & \text{for } \varnothing \subsetneqq E \subsetneqq S \\ 1 & \text{for } E = S \end{cases} .$$

A neo-additive capacity is additive on non-extreme outcomes, hence the name. We interpret the additive part of a neo-additive capacity as follows: p represents the agent's beliefs about the likelihood of uncertain events and $1 - \delta$ represents the agent's degree of confidence in this belief. The complement of the degree of confidence is the degree of ambiguity δ.

It can be shown that the Choquet integral of a simple act f with respect to a neo-additive capacity ν based on p is given by

$$\int f d\nu = \delta\alpha x_n + \delta(1-\alpha)x_1 + (1-\delta)\sum_{i=1}^{n} x_i p(\{s \in S | f(s) = x_i\}),$$

where f takes on the values $x_1 \geq \cdots \geq x_n$ (Chateauneuf et al. 2007). Thus, with respect to a neo-additive capacity, the Choquet integral of a simple act f is the weighted sum of the worst outcome under f, the best outcome under f, and the expected value of f with respect to p.

It follows that the Choquet expected utility of the simple act f with respect to

[36]Note that the composition $u(f(s)) : S \to \mathbb{R}$ is a simple act.

the neo-additive capacity ν based on p is given by

$$V_p(f) := \int u(f)d\nu = \delta\alpha u(x_n) + \delta(1-\alpha)u(x_1) + (1-\delta)E_p(f),$$

where $E_p(f) \equiv \sum_{i=1}^{n} u(x_i)p(\{s \in S | u(f(s)) = u(x_i)\})$. That is, it is the weighted sum of the minimum utility under f, the maximum utility under f, and the expected utility of f with respect to p. We interpret α as the degree of pessimism and $1-\alpha$ as the degree of optimism because they determine the respective weights given to the minimum utility and the maximum utility. As stated above, we interpret $1-\delta$ as the degree of confidence in p and δ as the degree of ambiguity. Note that in the absence of ambiguity ($\delta = 0$) Choquet expected utility reduces to expected utility.

A.1.2 The Model

Placing the model in this framework, the state space is $S = \{accident, no\ accident\}$. The outcome space is $X = \{m(c),\ M(c)\}$, where $m(c)$ and $M(c)$ depend on the applicable liability rule. Because a negligence rule effectively imposes no liability if $c \leq \bar{c}$ and strict liability if $c > \bar{c}$, we only need to define $m(c)$ and $M(c)$ for the rules of no liability and strict liability. Under a rule of no liability, $m(c) = M(c) = k - c$. Under strict liability, $m(c) = k - c - l(c)$ and $M(c) = k - c$. The set of acts is $\mathcal{F} = \{f(c) : c \geq 0\}$, where $f(c) = \begin{cases} m(c) & \text{if } s = accident \\ M(c) & \text{if } s = no\ accident \end{cases}$. The injurer's Bernoulli utility function is $u(f(c)) = f(c)$.

The injurer's beliefs about accident risk are given by a neo-additive capacity ν based on the probability distribution $p = \{\pi(c), 1 - \pi(c)\}$. With a slight abuse of notation, we say ν is based on π and we define $\nu(\pi) := \nu(accident)$, $\nu(1 - \pi) := \nu(no\ accident)$, and $V_\pi(c) := V_p(f(c))$. In addition, we define $E_\pi(c) := \pi(c)m(c) + (1 - \pi(c))M(c)$. It follows that the injurer's Choquet expected utility of exercising level of care c under a rule of no liability or strict liability is $V_\pi(c) = \delta\alpha m(c) + \delta(1-\alpha)M(c) + (1-\delta)E_\pi(c)$ and under a negligence rule is $\begin{cases} V_\pi(c) \text{ under no liability} & \text{if } c \geq \bar{c} \\ V_\pi(c) \text{ under strict liability} & \text{if } c < \bar{c} \end{cases}$.

A.1.3 Relationship to Other Models

A principal advantage of Choquet expected utility with a neo-additive capacity is that it contains as special cases or is mathematically equivalent to several alternative models of decision making under uncertainty. Special cases include (see, e.g., Schipper 2005):

- subjective expected utility, if $\delta = 0$ and $\alpha \in [0, 1]$;
- maxmin expected utility (Wald 1950), if $\delta = 1$ and $\alpha = 1$;
- maxmax expected utility, if $\delta = 1$ and $\alpha = 0$; and
- Hurwicz (1951) criterion, if $\delta = 1$ and $\alpha \in [0, 1]$.

Additional special cases include:

- maxmin expected utility with multiple priors (Gilboa and Schmeidler 1989), if $\delta \in [0, 1]$ and $\alpha = 1$; and
- maxmax expected utility with multiple priors, if $\delta \in [0, 1]$ and $\alpha = 0$,

in each case where the set of priors is $\mathcal{D} = \{q \in \triangle : q \geq (1-\delta)p\}$ and p denotes the probability distribution on which the neo-additive capacity is based (see Eichberger and Kelsey 2009).

With a neo-additive capacity μ based on p, Choquet expected utility is mathematically equivalent to:

- subjective expected utility where the subjective probability of the least favorable event W is $q_W = (1-\delta)p_W + \delta\alpha$, the most favorable event B is $q_B = (1-\delta)p_B + \delta(1-\alpha)$, and every other event E is $q_E = (1-\delta)p_E$;
- α-maxmin expected utility with multiple priors (Ghirardato et al. 2004) where the set of priors is $\mathcal{D} = \{q \in \triangle : q \geq (1-\delta)p\}$ (see Chateauneuf et al. 2007; Eichberger and Kelsey 2009);
- rank dependent expected utility with probability weighting function $\omega(p) = \mu(p)$ (see Wakker 1990); and
- cumulative prospect theory with probability weighting function $\omega(p) = \mu(p)$ and symmetric treatment of gains and losses (see Tversky and Wakker 1995).

A.2 SOCIAL PLANNER'S PROBLEM

Here we prove the assertion, made in footnote 28, that c^* is the level of care that would be chosen by a rational social planner as part of a Pareto optimal allocation. Let w and v denote the initial wealth of the injurer and the victim, respectively. Let w_a and v_a denote the wealth of the injurer and the victim, respectively, in the event of an accident and let w_n and v_n denote the wealth of the injurer and the victim, respectively, in the event of no accident. The social planner's problem is

$$\max_{\substack{c\geq 0 \\ w_a, w_n, v_a, v_n \geq 0}} \quad \pi(c)v_a + (1-\pi(c))v_n$$

subject to

$$\delta\alpha w_a + \delta(1-\alpha)w_n + (1-\delta)[\pi(c)w_a + (1-\pi(c))w_n] = \bar{V}$$

and

$$[\pi(c)v_a + (1-\pi(c))v_n] + [\pi(c)w_a + (1-\pi(c))w_n] + c + L(c) = w + k + v.$$

The foregoing expresses the social planner's problem as maximizing the expected utility of the victim subject to meeting a required Choquet expected utility level for the injurer $\bar{V}$ and a resource constraint in which expected resource use equals the available resources. Expected resource use is calculated using objective accident risk because the social planner is rational. The solution to the social planner's problem is a Pareto optimal allocation. We may assume, without loss of generality, that $w_a = w_n$. With this assumption, the social planner's problem reduces to $\max_{c \geq 0}$ $w + k + v - \bar{V} - c - L(c)$, which is equivalent to choosing $c \geq 0$ to minimize total accident costs $c + L(c)$.

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CHAPTER 2

ALLOCATION RULES AND THE STABILITY OF MASS TORT CLASS ACTIONS

A class action allows one or more representative parties to sue or be sued on behalf of a class of similarly situated persons. Rule 23(b)(3) of the United States Federal Rules of Civil Procedure permits a case to proceed as a class action when, inter alia, "a class action is superior to other available methods for fairly and efficiently adjudicating the controversy" (Fed. R. Civ. P. 23(b)(3)). In light of the general tradeoff between equity and efficiency in matters of public policy and law (Okun 1975; Kaplow and Shavell 2002), a class action that satisfies the requirements of Rule 23(b)(3) would appear to be socially desirable.

A distinguishing feature of a Rule 23(b)(3) class action is that the putative class members have the right to opt out of the class action and pursue their own interests (Fed. R. Civ. P. 23(c)(2)(B)). Advocates of opt-out rights offer various deontological and instrumental arguments in their favor.[1] Notwithstanding the merits of such arguments, the right to opt out of a Rule 23(b)(3) class action creates the risk that the class will unravel in spite of the fact that a class action is in society's best interests. This risk is acknowledged widely among class action scholars (e.g., Abraham 1987; Mullenix 1991; Perino 1997; Rosenberg 2002), including by opt-out rights advocates (e.g., Schuck 1995; Rutherglen 1996; Nagareda 2003).

The risk that the class will unravel is thought to be particularly significant in the case of a Rule 23(b)(3) mass tort class action in which separate actions are

[1] The deontological arguments usually emphasize the concept of plaintiff autonomy and invoke the notion that "everyone should have his own day in court" (*Ortiz v. Fireboard Corp.*, 527 U.S. 815, 846 (1999)). The instrumental arguments usually emphasize the idea that the right to opt out serves as a mechanism to mitigate the principal-agent problems inherent in class actions. For summaries of these arguments and concise reviews of the literature, see, e.g., Perino (1997) and Eisenberg and Miller (2004b). For criticisms of these arguments, see, e.g., Rosenberg (2003).

viable (Coffee 1987; Bone 2003).[2] Indeed, there is empirical evidence that opt-out rates are highest in mass tort cases (Eisenberg and Miller 2004b). In recognition of the problem, several legal commentators propose restricting or even abolishing opt-out rights in mass tort class actions (e.g., Mullenix 1986; Coffee 1987; Rosenberg 2003). Contrary to such proposals, however, recent amendments to Rule 23 have expanded opt-out rights.[3]

One reason why the class might unravel in a Rule 23(b)(3) mass tort class action is adverse selection due to damage averaging (Coffee 1987; Bone 2003). Damaging averaging occurs when the allocation rule governing the division of the net recovery of the class among its members assigns class members with below-average (above-average) claims more (less) than their pro rata shares.[4] If the governing allocation rule engages in damage averaging, then, even if the per member expected recovery in the class action exceeds the mean expected recovery from separate actions, which may be the case if, for example, the class action enjoys economies of scale, superior prospects of prevailing at trial, or enhanced bargaining power in settlement negotiations, the amount that one or more putative class members with above-average claims can expect to recover by opting out may exceed the amount that they can expect to recover by remaining in the class action.

This chapter formally examines how different allocation rules influence the risk

[2] There is no single, universally accepted definition of "mass tort" litigation. Deborah Hensler, a leading class action scholar, defines it as "large scale personal injury or property damage litigation arising out of product use or exposure" (Hensler 2001, pp. 181-182). The American Bar Association Commission on Mass Torts defines it as involving "at least 100 civil tort actions arising from a single accident or use of or exposure to the same product or substance, each of which involves a claim in excess of $50,000 for wrongful death, personal injury or physical damage to or destruction of tangible property" (Willging 1999, pp. 8-9). Examples of high profile mass tort class actions include the Agent Orange litigation, the Dalkon Shield litigation, and several asbestos cases (Coffee 1995; Weinstein 1995).

[3] In 2003, Rule 23 was amended to explicitly authorize the court to refuse to approve a settlement in a Rule 23(b)(3) class action unless it affords class members a second opportunity to opt out after the terms of the settlement are known (Fed. R. Civ. P. 23(e)(3); Advisory Committee's Notes to Rule 23).

[4] According to Silver and Baker (1998, p. 1481), "[a]llocation plans used in class actions inevitably involve some degree of damage averaging."

that the class will unravel in a Rule 23(b)(3) mass tort class action. I focus on three allocation rules: (1) equal sharing; (2) pro rata by damage claims; and (3) pro rata by outside options (i.e., expected claim values). I consider these rules for two reasons. First and foremost, they run the gamut of damage averaging. Rules 1 and 3 correspond to full damage averaging and no damage averaging, respectively, while rule 2 involves partial damage averaging.[5] Second, these rules are natural and obvious candidates for "fair" allocation standards; as one commentator states in a closely related context, each rule has "immediate, though perhaps naive, appeal" (Kornhauser 1998, p. 1568).[6]

I analyze a two-stage model of class action formation in which a single defendant faces multiple plaintiffs with heterogeneous damage claims. A global class action is certified at the outset. In stage 1, the plaintiffs play a coalition formation game in which each plaintiff simultaneously announces whether it will remain in the class or opt out. Stage 1 is modeled as a noncooperative game in partition function form (see, e.g., Bloch 2003; Yi 2003). The global class is *stable* if the strategy profile in which all plaintiffs remain in the class constitutes a pure strategy Nash equilibrium of the game. In stage 2, the class action and any individual actions by opt-out plaintiffs are resolved via either litigation or settlement. Stage 2 is modeled in the divergent expectations tradition (see, e.g., Priest and Klein 1984) and assumes that if the parties settle their dispute they divide the joint surplus from settlement according to the asymmetric Nash bargaining solution.

[5] Rule 2 involves partial damage averaging in my model because plaintiffs share a common probability of prevailing at trial. It also would involve partial damage averaging if the probability of prevailing at trial were higher for plaintiffs with above-average claims than for plaintiffs with below-average claims. However, if the probability of prevailing at trial were lower for plaintiffs with above-average claims than for plaintiffs with below-average claims, then rule 2 would involve negative averaging whereby class members with below-average (above-average) claims would receive less (more) than their pro rata shares.

[6] Rule 3 reflects the normative standard embraced by many class action scholars (Silver 2000), and arguably by the United States Supreme Court (Rosenberg 2003). For a forceful economic argument in support of rule 2, see Rosenberg (2002).

I examine the *asymptotic stability* of the global class under each allocation rule. The global class is asymptotically stable if the probability that it is stable converges to one as the number of plaintiffs becomes arbitrarily large. I am interested in the asymptotic stability of the global class because in the situation under consideration—a Rule 23(b)(3) mass tort class action—the number of plaintiffs presumably is large. This presumption follows not only from the fact that it is a *mass* tort class action, but also because certification under Rule 23(b)(3) implies that the class is "numerous" (Fed. R. Civ. P. 23(a)(1)).

I show that the global class is asymptotically stable if the net recovery of the class will be allocated pro rata in accordance with its members' outside options (rule 3), but that it may not be asymptotically stable if the net recovery of the class will be shared equally (rule 1) or allocated pro rata in accordance with the members' damage claims (rule 2). For rules 1 and 2, I derive necessary and sufficient conditions for the asymptotic stability of the global class. I also derive sufficient conditions for the asymptotic stability and instability of the global class under rules 1 and 2. In addition, I show that the asymptotic stability of the global class under rule 1 necessarily implies the asymptotic stability of the global class under rule 2 but not vice versa.

I find that a key determinant of the asymptotic stability of the global class under rule 1 is the shape of the distribution of the plaintiffs' damage claims. Generally speaking, the global class is more likely to be asymptotically stable under rule 1 if the expected damage claim is high and the range of damage claims is narrow. If the claims distribution is unimodal and has a bounded support, this implies that the global class is more likely to be asymptotically stable under rule 1 when the distribution is negatively skewed. In addition, I find that the magnitude of the scale benefits of the class action and the plaintiffs' probability of prevailing at trial and bargaining power in settlement negotiations are important determinants of the asymptotic stability of the global class under rules 1 and 2. In particular, if the scale benefits of a class action are high, the global class is more likely to be asymptotically stable under rules 1 and 2 if the plaintiffs' bargaining power in settlement negotiations is low. If, however, the scale benefits of a class action are low, the global class is less likely to be asymptotically stable under rules 1 and 2 if the plaintiffs' probability of prevailing at trial is high or their bargaining power in settlement negotiations is low.

In an effort to understand the relative stability of the global class under the three allocation rules, I simulate the model using standard Monte Carlo methods. As compared to rule 3, I find that the global class is asymptotically stable about two-thirds as often under rule 2 and about a quarter as often under rule 1. The simulations also confirm my findings regarding the determinants of class stability.

An important implication of my results is that selecting an allocation rule in a Rule 23(b)(3) mass tort class action generally involves a tradeoff between ex ante and ex post efficiency. On the one hand, the risk that the class will unravel due to adverse selection generally increases with the degree of damage averaging in which the governing allocation rule engages. On the other hand, the cost of implementing an allocation rule generally decreases as the degree of damage averaging in which it engages increases (Coffee 1987, 1998; Silver and Baker 1998; Silver 2000).[7] At the same time, however, my results suggest when this tradeoff may be avoided, e.g., when the plaintiffs' damage claims are severely negatively skewed over a very narrow range or when a class action would achieve significant scale economies and the likelihood that the plaintiffs will prevail on the merits is low.

More generally, my results provide guidance regarding when and how allocation

[7] As Silver (2000, p. 226) explains, "[i]t is more expensive to pay claimants amounts that roughly reflect the size and strength of their claims than it is to engage in damages averaging and pay them equal amounts, and it is more expensive still to distribute payments that reflect fine differences between claimants."

rules may be used to promote the stability of the class in Rule 23(b)(3) mass tort class actions. Accordingly, they suggest criteria to attorneys and courts for structuring and approving efficient allocations plans in such actions; e.g., if the proposed allocation plan is likely to destabilize the class then perhaps the court should not find that the settlement is "fair, reasonable, and adequate" (Fed. R. Civ. P. 23(e)(1)(C)). The results also suggest criteria for class certification under Rule 23(b)(3); e.g., if no cost-effective allocation rule exists under which the global class would be asymptotically stable then perhaps a class action is not "superior to other available methods for . . . efficiently adjudicating the controversy" (Fed. R. Civ. P. 23(b)(3)).

The remainder of the chapter proceeds as follows. Section 2.1 briefly discusses the institutional background and related literature. Section 2.2 presents the model. Section 2.3 analyzes the asymptotic stability of the global class under each allocation rule. Section 2.4 presents the results of the Monte Carlo simulations. Section 2.5 contains concluding remarks. It discusses implications and possible extensions of the model. The Appendix contains certain mathematical details.

2.1 INSTITUTIONAL BACKGROUND AND RELATED LITERATURE

2.1.1 Introduction to Class Actions and Rule 23

The class action is a procedural device pursuant to which "[o]ne or more members of a class may sue or be sued as representative parties on behalf of all members" (Fed. R. Civ. P. 23(a)). In general, the resolution of a class action binds all members of the class, including absent parties.[8] Thus, the class action forms an exception to the "principle of general application in Anglo-American jurisprudence that one is not bound by a judgment . . . in a litigation in which he is not designated

[8]For an historical analysis of the binding effect of class actions on absent parties, see Hazard et al. (1998).

as a party or to which he has not been made a party by service of process" (*Ortiz v. Fireboard Corp.*, 527 U.S. 815, 846 (1999)).

The raison d'être of the class action is efficiency. Class actions can enhance efficiency in several ways. A class action can solve a collective action problem in a case in which individual actions are not economically viable, thereby promoting optimal deterrence (*Amchem Products Inc. v. Windsor*, 521 U.S. 591, 617 (1997); Macey and Miller 1991). When individual actions are economically viable, a class action can achieve economies of scale, thereby reducing litigation costs and promoting optimal investment in the litigation (Hay and Rosenberg 2000), and promote uniformity in the law, thereby avoiding the social costs associated with legal inconsistency.

The historical roots of the class action run deep. Litigation by representatives of a group seeking to redress communal harms dates back medieval England (Yeazell 1987). The modern ancestry of the class action includes the bill of peace with multiple parties, which was developed in the seventeenth century by the Court of Chancery in England (Chafee 1932, 1950). In the United States, the first provision for class actions in federal courts, Rule 48 of the Federal Equity Rules, was adopted in 1843.[9] It permitted a representative suit when the parties on either side were too numerous to be brought before the court without manifest inconvenience and oppressive delays and the representative parties were sufficient to represent the interests of the absent parties (42 U.S. (1 How.) lvi (1843)).[10] In 1912, Rule 48

[9]Rule 48 provided:

> Where the parties on either side are very numerous, and cannot, without manifest inconvenience and oppressive delays in the suit, be all brought before it, the court in its discretion may dispense with making all of them parties, and may proceed in the suit, having sufficient parties before it to represent all the adverse interests of the plaintiffs and the defendants in the suit properly before it. But in such cases the decree shall be without prejudice to the rights and claims of all the absent parties.

[10]Note that although Rule 48 provided for representative suits, its last sentence enigmatically provided that the suit's resolution would not bind absent parties. A decade after the adoption

was amended and restated as Rule 38. The revised rule succinctly provided, "When the question is one of common or general interest to many persons constituting a class so numerous as to make it impracticable to bring them all before the court, one or more may sue or defend for the whole" (226 U.S. 659 (1912)).

The existing class action device in the United States is Rule 23 of the Federal Rules of Civil Procedure. Originally adopted in 1938, Rule 23 was substantially revised in 1966 and last amended in 2007. As amended, Rule 23(a) enumerates four prerequisites to a class action: "(1) the class is so numerous that joinder of all members is impracticable; (2) there are questions of law or fact common to the class; (3) the claims or defenses of the representative parties are typical of the claims or defenses of the class; and (4) the representative parties will fairly and adequately protect the interests of the class." Commonly known as numerosity, commonality, typicality, and adequacy of representation (*Amchem Products Inc. v. Windsor*, 521 U.S. 591, 613 (1997)), these prerequisites echo the requirements of the former equity rules (Hensler et al. 2000).

Rule 23(b) specifies three situations in which a case that satisfies the prerequisites of Rule 23(a) may proceed as a class action. Rule 23(b)(1) permits a class action when separate actions would create a risk that the party opposing the class would face inconsistent or varying adjudications or that an adjudication as to one or more class members would prejudice the interests of other class members. For example, Rule 23(b)(1) traditionally includes "limited fund" cases (*Ortiz v. Fireboard Corp.*, 527 U.S. 815, 834 (1999)), in which "claims are made by numerous persons against a fund insufficient to satisfy all claims" (Advisory Committee's Notes to Rule 23). Rule 23(b)(2) covers situations where the actions or omissions

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of Rule 48, the Supreme Court ignored the rule's enigmatic proviso and indicated in dicta that when "a court of equity permits a portion of the parties in interest to represent the entire body, . . . the decree binds all of them the same as if all were before the court" (*Smith v. Swormstedt*, 57 U.S. (1 How.) 288, 303 (1853)).

of the party opposing the class affect the entire class and injunctive or declaratory relief respecting the class as a whole is appropriate. A prime example is a civil rights suit alleging unlawful discrimination against a class (Advisory Committee's Notes to Rule 23; *Amchem Products Inc. v. Windsor*, 521 U.S. 591, 614 (1997); see also Miller 1979).

Rule 23(b)(3) provides that a class action may be maintained if common questions of law or fact predominate over individual questions and if a class action is "superior to other available methods for fairly and efficiently adjudicating the controversy." According to its drafters, Rule 23(b)(3) "encompasses those cases in which a class action would achieve economies of time, effort, and expense, and promote uniformity of decision as to persons similarly situated, without sacrificing procedural fairness or bringing about other undesirable results" (Advisory Committee's Notes to Rule 23). Rule 23(b)(3) is a catchall for class actions that do not fit into the "pigeonholes" of Rule 23(b)(1) or (2) (Bronsteen and Fiss 2003, p. 1434), but that "may nevertheless be convenient and desirable depending upon the particular facts" (Advisory Committee's Notes to Rule 23).

Class actions maintained under Rule 23(b)(1) and Rule 23(b)(2) are mandatory; class members do not have a statutory right to exclude themselves from the class (Fed. R. Civ. P. 23(c)(3)(A)). By contrast, putative class members have the right to opt out of a Rule 23(b)(3) class action. The rules require the court to exclude from the class any member who requests exclusion in accordance with the time and manner restrictions set forth in the class action notice (Fed. R. Civ. P. 23(c)(2)(B)(v)–(vi)). Those who duly opt out are not bound by the outcome of the class action (Fed. R. Civ. P. 23(c)(3)(B)).

When deemed appropriate by the court, a class may be divided into subclasses and each subclass treated as a separate class (Fed. R. Civ. P. 23(c)(4)(B)). In addition, the court may limit the scope of a class action to one or more particular issues (Fed. R. Civ. P. 23(c)(4)(A)). In a products liability case, for example, the court may certify a class action only on the issue of the defendant's liability and require that the class members proceed individually to prove the amounts of their respective damage claims (see Advisory Committee's Notes to Rule 23).

Any settlement of a certified class action must be approved by the court (Fed. R. Civ. P. 23(e)(1)(A)). In order for the settlement to be binding, the court must conduct a hearing and find that the settlement is "fair, reasonable, and adequate" (Fed. R. Civ. P. 23(e)(1)(C)). Although the rules do not define these terms, courts have elaborated several multifactor tests (see generally Macey and Miller 2009). The court's review of a proposed settlement is distinct from and in addition to the court's certification inquiry, including with respect to the adequacy requirement and, in the case of a Rule 23(b)(3) class action, the superiority requirement (*Amchem Products Inc. v. Windsor*, 521 U.S. 591, 619-22 (1997); Nagareda 2002). The court may refuse to approve a proposed settlement of a Rule 23(b)(3) class action unless it affords class members a new opportunity to opt out after the terms of the settlement are known (Fed. R. Civ. P. 23(e)(3); Advisory Committee's Notes to Rule 23). In addition, the rules provide that any class member may object to a proposed settlement and that any such objection may be withdrawn only with the court's approval (Fed. R. Civ. P. 23(e)(4); Leslie 2007).

When class actions are certified in mass tort cases, they usually are certified under Rule 23(b)(3) (Weinstein 1995). However, the use of class actions to resolve mass tort cases is highly controversial (Schuck 1995; Hensler 2001). Moreover, judicial attitudes towards certification of mass tort class actions have ebbed and flowed since the 1966 overhaul of Rule 23 (Coffee 1995; Schuck 1995; Weinstein 1995; Perino 1997; Tidmarsh 1998).[11]

[11] The advisory notes accompanying the 1966 amendments to Rule 23 state that a "'mass accident' resulting in injuries to numerous persons is ordinarily not appropriate for a class action because of the likelihood that significant questions, not only of damages but of liability and

2.1.2 *Relation to the Literature*

This chapter relates to several strands of literature within law, economics, and their intersection. Within law, this chapter contributes to the vast literature on class actions. Silver (2000) surveys this literature. In particular, this chapter adds to the legal scholarship that discusses allocations in class actions, including Morawetz (1993), Silver and Baker (1997, 1998), Coffee (1998), Dana (2006), Edelman et al. (2006), and Macey and Miller (2009).

This chapter closely relates and directly contributes to the small, but growing literature on the economics of class actions, which includes Kornhauser (1983, 1998), Che (1996), Perino (1997), Marceau and Mongrain (2003), and Deffains and Langlais (2009). Kornhauser (1983, 1998) and Perino (1997) model the formation of a class action as a cooperative game in characteristic function form. Kornhauser considers an "allocation of common costs" game and adopts the core of the game as the standard for a "fair" allocation. While certain of his results are comparable to results in this chapter, Kornhauser focuses on how different court procedures for approving settlements (intervention rules, voting rules, and attorney compensation schemes) influence whether the class attorney and the defendant will propose a fair allocation. Perino uses a simple, three-player game to construct a series of examples that illustrate how the concept of core stability can elucidate several academic theories and real-world phenomena pertaining to class actions and opt-out rights. Although he does not develop a general model, Perino demonstrates the usefulness of core theory for the analysis of class action dynamics.

Che (1996), Marceau and Mongrain (2003), and Deffains and Langlais (2009) study the equilibrium formation of class actions using different noncooperative

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defenses to liability, would be present, affecting the individuals in different ways" (Advisory Committee's Notes to Rule 23). However, courts often disregard this comment (Wright et al. 2005, sec. 1783) and ocassionally expressly repudiate it (see, e.g., *In re A.H. Robins Co., Inc.*, 880 F.2d 709, 729-38 (4th Cir. 1989)).

games. Che examines the adverse selection hypothesis in a model that features a single defendant, two types of plaintiffs (small stakes and large stakes), and full damaging averaging. He focuses on the role of asymmetric information, considering two cases: when the defendant has complete information about the plaintiffs' claims and when the plaintiffs' claims are private information. In both cases he finds equilibria in which the class partially or fully unravels, although he finds that pure adverse selection arises only in the case of complete information. Che's model is closely related to my model and his results in the case complete information are comparable to my results on class stability under equal sharing. Marceau and Mongrain analyze a waiting game among multiple plaintiffs with heterogenous damage claims and examine how the degree of damage averaging influences which plaintiff will assume the role of class representative and initiate the class action. They find that if there is full damage averaging, the class representative will be the plaintiff with the lowest damage claim, while if there is less than full damage averaging, other plaintiffs may initiate the class action. Deffains and Langlais consider a sequential entry game between two plaintiffs (high stakes and low stakes) that have been injured by the same defendant. They focus on the consequences of information externalities and information sharing for the formation of a class action, though in an extension of their model they prove one result on damage averaging that is comparable to results in this chapter.

In addition, this chapter draws on the litigation and settlement literature within law and economics, including, most notably, Landes (1971), Posner (1973), Gould (1973), Shavell (1982), Priest and Klein (1984), and Hylton (2006). A recent survey of this literature is Spier (2007). This chapter also draws on and contributes an application to the literature within economics and game theory on noncooperative games of coalition formation. Surveys of this literature are provided by Konishi et al. (1997), Bloch (1997, 2003), and Yi (2003).

2.2 TWO-STAGE MODEL OF CLASS ACTION FORMATION

Consider a mass tort case involving n plaintiffs and one defendant. Let N denote the set of all plaintiffs and i denote an arbitrary plaintiff in N. All parties are risk neutral expected wealth maximizers.

Each plaintiff $i \in N$ has a damage claim θ_i against the defendant. Each plaintiff's damage claim is its private knowledge. However, it is common knowledge that the plaintiffs' damage claims are independent and identically distributed according to a cumulative distribution function F_θ and a probability density function f_θ that is strictly positive on its support set $[\underline{\theta}, \overline{\theta}]$, where $0 < \underline{\theta} < \overline{\theta} < \infty$. I assume that the defendant's assets are available and sufficient to satisfy the damage claims of all plaintiffs.[12]

At the outset, a class action on behalf of all plaintiffs is certified under Rule 23(b)(3). In stage 1, each plaintiff simultaneously announces whether it will remain in the class action or opt out pursuant to Rule 23(c).[13] Let $A \subseteq N$ denote the subset of plaintiffs that remain in the class action and let N/A denote the subset of plaintiffs that opt out. I refer to A as the *class*, to each plaintiff $i \in A$ as a *class member*, and to each plaintiff $i \in N \backslash A$ as an *opt-out plaintiff*. The number of class members is denoted by $|A|$ and I refer to $|A|$ as the *class size*.

Following Che (1996), I assume that each plaintiff's announcement is binding (i.e., no class member may opt out and no opt-out plaintiff may rejoin the class) and that each opt-out plaintiff must pursue its claim individually (e.g., no other class actions are maintained on behalf of opt-out plaintiffs and no opt-out plaintiffs maintain joinder actions under Rule 20).[14] Accordingly, the plaintiffs' announce-

[12] I revisit this and other key assumptions of the model in Section 2.5.

[13] Assuming simultaneous announcements captures the idea that a plaintiff does not know the announcements of the other plaintiffs when it makes its announcement.

[14] Rule 20 provides, in pertinent part: "All persons may join in one action as plaintiffs if they assert any right to relief jointly, severally, or in the alternative in respect of or arising out of the same transaction, occurrence, or series of transactions or occurrences and if any question of law

ments induce a partition Ω^A of N, where $\Omega^A = \left\{A, (i)_{i \in N \setminus A}\right\}$. I refer to Ω^A as the *class structure* and to each $\omega \in \Omega^A$ as a *stage 2 plaintiff*. For the sake of brevity, I often refer to a stage 2 plaintiff simply as a plaintiff.

In stage 2, the defendant and each plaintiff $\omega \in \Omega^A$ resolve their dispute via either litigation or settlement. I assume that the class' damage claim equals the expected damage claim multiplied by the class size: $\theta_A = E[\theta] \cdot |A|$. In addition, I assume that there are no externalities or spillovers across plaintiffs. In particular, I assume that the class action and any individual actions by opt-out plaintiffs are resolved simultaneously and that all plaintiffs' claims have the same priority in bankruptcy.

Because the plaintiffs' expected payoffs in stage 1 are functions of their expected recoveries in stage 2, I proceed in reverse order and begin with stage 2.

2.2.1 Stage 2: Dispute Resolution

A. Probability of Settlement

Plaintiff ω and the defendant settle rather than litigate their dispute if a settlement range exists—i.e., if plaintiff ω's reservation price (its minimum settlement demand) is less than or equal to the defendant's reservation price (its maximum settlement offer):

$$P_\omega \theta_\omega - C_\omega \leq Q_\omega \theta_\omega + K_\omega, \tag{2.1}$$

where (i) P_ω and Q_ω denote the respective estimates by plaintiff ω and the defendant of the probability that plaintiff ω would prevail at trial and (ii) $C_\omega > 0$ and $K_\omega > 0$ denote the respective litigation costs of plaintiff ω and the defendant.[15] If no settlement range exists the parties litigate. Condition (2.1) implicitly assumes

or fact common to all these persons will arise in the action" (Fed. R. Civ. P. 20(a)).

[15] Condition (2.1) is a so-called Landes-Posner-Gould condition (see Landes 1971; Posner 1973; Gould 1973; Hylton 2006).

that if the parties litigate and plaintiff ω prevails at trial the defendant is liable to plaintiff ω for its damage claim θ_ω,[16] that the parties bear their own litigation costs,[17] and that settlement costs are zero.[18] Note that by (2.1), $Q_\omega > P_\omega$ is a sufficient (but not necessary) condition for settlement and $P_\omega > Q_\omega$ is a necessary (but not sufficient) condition for litigation.

The parties estimate the probability that plaintiff ω would prevail at trial with error. In particular, I assume that the parties' estimates are given by

$$P_\omega = W_\omega + \epsilon_\omega; \tag{2.2}$$

$$Q_\omega = W_\omega + \mu_\omega, \tag{2.3}$$

where (i) W_ω is the probability that plaintiff ω would prevail at trial and (ii) ϵ_ω and μ_ω represent the respective prediction errors of plaintiff ω and the defendant. I assume that ϵ_ω and μ_ω are independently realized at the beginning of stage 2 and that each is uniformly distributed on the interval $[\max\{-W_\omega, W_\omega - 1\}, \min\{W_\omega, 1 - W_\omega\}]$. The latter assumption ensures that the parties' estimates of the probability that plaintiff ω would prevail at trial are between zero and one ($P_\omega \in [0,1]$ and $Q_\omega \in [0,1]$) and are correct in expectation ($E[P_\omega] = E[Q_\omega] = W_\omega$). As Figure 2.1 displays, this assumption also implies that the variance of the parties' prediction errors is zero at $W_\omega = 0$ and $W_\omega = 1$, when the outcome of a trial is certain, and achieves its maximum at $W_\omega = \frac{1}{2}$, when the outcome of a trial is most uncertain (cf. Priest and Klein 1984; Hylton 2006).

Given (2.2) and (2.3), we can restate condition (2.1) as follows: plaintiff ω and

[16] That is, I assume the court accurately determines plaintiff ω's damages and awards compensatory but not punitive damages. I follow Che (1996) in making this assumption.

[17] This reflects the American rule (see, e.g., Shavell 1982; Hylton 1993)

[18] This is a standard assumption in the literature (see, e.g., Shavell 1982; Hylton 1993, 2006). Alternatively, we could relax this assumption and then assume that litigation costs exceed settlement costs, in which case C_ω and K_ω would denote the excess of litigation costs over settlement costs for plaintiff ω and the defendant, respectively.

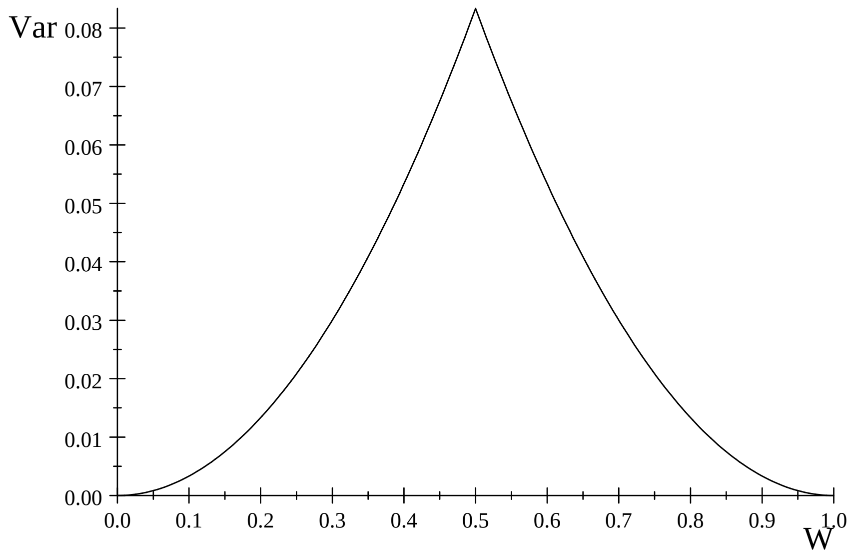


Figure 2.1: Variance of ϵ_ω and μ_ω as a Function of W_ω

the defendant settle rather than litigate their dispute if

$$\epsilon_\omega - \mu_\omega \leq \frac{C_\omega + K_\omega}{\theta_\omega}. \tag{2.4}$$

It follows that the probability that plaintiff ω and the defendant settle is

$$\phi_\omega = F_{\Delta(W_\omega)}\left(\frac{C_\omega + K_\omega}{\theta_\omega}\right), \tag{2.5}$$

where $F_{\Delta(W_\omega)}$ is the cumulative distribution function of $\Delta(W_\omega) = \epsilon_\omega - \mu_\omega$.[19] Conversely, the probability that plaintiff ω and the defendant litigate is $1 - \phi_\omega$.

We can infer from (2.4) and (2.5) how various factors generate litigation in the model. Condition (2.4) implies that litigation may result from "overoptimism" on the part of both parties (i.e., $\epsilon_\omega > 0$ and $\mu_\omega < 0$) (cf. Shavell 1982; Hylton 2006). Equation (2.5) implies that the probability that the parties litigate is weakly decreasing in joint litigation costs, $C_\omega + K_\omega$, and weakly increasing in the litigation

[19] The distribution of $\Delta(W_\omega)$ is derived in the Appendix.

stakes, θ_ω (cf. Posner 1973; Hylton 2006). This is because F_{Δ_ω} is nondecreasing. In addition, equation (2.5) implies that the probability of litigation is weakly greater the more uncertain is the outcome of a trial (cf. Priest and Klein 1984; Hylton 2006). This is because $F_{\Delta(W_\omega)}$ first-order stochastically dominates $F_{\Delta(W_\upsilon)}$ on $[0,1]$ if $\left|W_\omega - \frac{1}{2}\right| < \left|W_\upsilon - \frac{1}{2}\right|$.

B. Expected Recovery

If the parties litigate, plaintiff ω expects to recover

$$\pi_\omega^L = P_\omega \theta_\omega - C_\omega. \tag{2.6}$$

If the parties settle, plaintiff ω expects to recover its minimum settlement demand, $P_\omega\theta_\omega - C_\omega$, plus its bargained-for share of the joint surplus from settlement, $\Lambda_\omega = (Q_\omega\theta_\omega + K_\omega) - (P_\omega\theta_\omega - C_\omega)$. I assume that the parties divide the surplus Λ_ω in accordance with the asymmetric Nash bargaining solution.[20] Accordingly, if $\lambda_\omega \in [0,1]$ represents plaintiff ω's bargaining power, plaintiff ω expects to recover

$$\begin{aligned} \pi_\omega^S &= (P_\omega\theta_\omega - C_\omega) + \lambda_\omega \Lambda_\omega \\ &= \lambda_\omega (Q_\omega\theta_\omega + K_\omega) + (1-\lambda_\omega)(P_\omega\theta_\omega - C_\omega). \end{aligned} \tag{2.7}$$

Hence, plaintiff ω's *expected recovery* in stage 2 is

$$\begin{aligned} \pi_\omega &= \phi_\omega \pi_\omega^S + (1-\phi_\omega)\pi_\omega^L \\ &= \phi_\omega\lambda_\omega (Q_\omega\theta_\omega + K_\omega) + (1-\phi_\omega\lambda_\omega)(P_\omega\theta_\omega - C_\omega). \end{aligned} \tag{2.8}$$

[20] Formally, the bargaining problem is $\langle X_\omega, \mathbf{0}\rangle$, where $X_\omega = \{(x_\omega, x_d) \in \mathbb{R}_+^2 : x_\omega + x_d = \Lambda_\omega\}$ is the set of possible divisions and $\mathbf{0} = (0,0)$ is the disagreement point. If $\lambda_\omega \in [0,1]$ represents plaintiff ω's bargaining power, then the asymmetric Nash solution is $(x_\omega^*, x_d^*) = (\lambda_\omega\Lambda_\omega, (1-\lambda_\omega)\Lambda_\omega)$, which is the unique solution to $\max_{(x_\omega,x_d)\in X_\omega} (x_\omega)^{\lambda_\omega} (x_d)^{1-\lambda_\omega}$. For a discussion of the asymmetric Nash bargaining solution, see, e.g., Muthoo (1999).

This formulation of plaintiff ω's expected recovery relies on two presumptions. First, it presumes that plaintiff ω knows (or at least correctly estimates) the probability of settlement, ϕ_ω, but does not know the underlying data generating process (i.e., equations (2.2) and (2.3)), which seems reasonable insofar as the plaintiff (or its lawyer) has experience with or data on similar cases (cf. Priest and Klein 1984). Second, it presumes that the parties know (or at least learn) each other's reservation price. This is a common presumption in the literature (see, e.g., Bar-Gill 2006). To support this presumption in our setting, it is sufficient (and seems reasonable) to assume that, in stage 2, the defendant learns plaintiff ω's damage claim (while continuing to assume that the damage claim of each class member remains its private knowledge, unknown to the defendant, the other class members, and any opt-out plaintiffs) and each party learns the other party's estimate of the probability that the plaintiff would prevail at trial (without updating its own estimate).[21]

C. Additional Assumptions

Symmetry. On the basis that class certification under Rule 23(b)(3) implies that the plaintiffs are "similarly situated" with respect to their factual and legal claims against the defendant (Advisory Committee's Notes to Rule 23), I assume that: (i) each individual plaintiff has the same litigation costs and the defendant's litigation costs are the same with respect to each individual plaintiff:

$$C_\omega = C \text{ and } K_\omega = K \text{ for all } \omega \neq A; \tag{2.9}$$

$$C_A = C \text{ and } K_A = K \text{ for } |A| = 1; \tag{2.10}$$

[21] The no updating assumption relies on the parties' ignorance of the data generating process (i.e., equations (2.2) and (2.3)).

(ii) each plaintiff has the same probability that it would prevail at trial:

$$W_\omega = W \text{ for all } \omega \in \Omega^A\text{; and} \tag{2.11}$$

(iii) each plaintiff has the same bargaining power in settlement negotiations:

$$\lambda_w = \lambda \text{ for all } \omega \in \Omega^A. \tag{2.12}$$

Economies of scale. Class certification under Rule 23(b)(3) also implies that "a class action would achieve economies of time, effort, and expense" (Advisory Committee's Notes to Rule 23). Accordingly, I assume that although litigation costs are increasing in class size, per-plaintiff litigation costs are weakly decreasing in class size,[22] but always positive:

$$C_A \geq C \text{ and } K_A \geq K \text{ for } |A| > 1; \tag{2.13}$$

$$\frac{1}{|A|}C_A \leq \frac{1}{|A'|}C_{A'} \text{ and } \frac{1}{|A|}K_A \leq \frac{1}{|A'|}K_{A'} \text{ for all } A, A' \subseteq N,\ |A| > |A'|\,; \tag{2.14}$$

$$\frac{1}{|A|}C_A \to c > 0 \text{ and } \frac{1}{|A|}K_A \to k > 0 \text{ as } |A| \to \infty. \tag{2.15}$$

Because the class enjoys neither a higher probability of prevailing at trial nor enhanced bargaining power in settlement negotiations, these scale benefits provide the key incentive in the model for plaintiffs to remain in the class action.[23] Indeed, the per-plaintiff net recovery is weakly increasing in class size and achieves its maximum when the class includes all plaintiffs (i.e., when $A = N$).

[22] I follow Che (1996) in making this assumption, although he assumes that per-plaintiff litigation costs are strictly decreasing in class size.

[23] Scale economies are the basic force that attracts plaintiffs to the class action in Che's (1996) model as well.

Viability of litigation. Consistent with Che (1996), I restrict attention to mass tort cases in which litigation is objectively viable for each plaintiff:

$$W\underline{\theta} - C \geq 0. \tag{2.16}$$

Accordingly, the model does not pertain to mass torts for which a class action is socially desirable because it solves a collective action problem. Rather, it pertains to mass torts for which a class action is socially desirable because it achieves economies of scale. Similarly, I assume that litigation against each plaintiff is objectively viable for the defendant:

$$W\underline{\theta} - K \geq 0. \tag{2.17}$$

2.2.2 Stage 1: Class Formation Game

The formation of the class is modeled as a noncooperative, simultaneous move, single coalition formation game Γ, where: (i) the set of players is the set of all plaintiffs, N; (ii) the set of actions available to each plaintiff is $\{In, Out\}$; and (iii) payoffs are described by a per-member partition function $V = \mathcal{R} \circ \Pi$, where (a) Π is a partition function that assigns to each class structure Ω^A a vector $\pi \in \mathbb{R}^{|\Omega^A|}$ which specifies the expected recovery π_ω of each plaintiff $\omega \in \Omega^A$ in stage 2 and (b) $\mathcal{R}$ is an allocation rule that maps each stage 2 expected recovery profile π into a vector $v \in \mathbb{R}^n$ which specifies the expected payoff v_i of each plaintiff $i \in N$ at stage 1.

Under any allocation rule, the expected payoff of each opt-out plaintiff $i \in N \backslash A$ under class structure Ω^A is simply the expected value at stage 1 of its expected recovery in stage 2. However, the expected payoff of each class member $i \in A$ under class structure Ω^A depends on the allocation rule $\mathcal{R}$. I consider the following three allocation rules, each of which is defined in terms of the vector of expected payoffs v it induces:

$$(\mathcal{R}^1) \quad \textit{Equal sharing}\text{: } v_i(\Omega^A) = \begin{cases} \frac{1}{|A|} E\left[\pi_A\right] & \text{for } i \in A \\ E\left[\pi_i\right] & \text{for } i \in N \backslash A \end{cases};$$

$$(\mathcal{R}^2) \quad \textit{Pro rata by damage claims}\text{: } v_i(\Omega^A) = \begin{cases} \frac{\theta_i}{\theta_A} E\left[\pi_A\right] & \text{for } i \in A \\ E\left[\pi_i\right] & \text{for } i \in N \backslash A \end{cases};$$

$$(\mathcal{R}^3) \quad \textit{Pro rata by outside options}\text{: } v_i(\Omega^A) = \begin{cases} \frac{E[\pi_i]}{\sum_{j \in A} E[\pi_j]} E\left[\pi_A\right] & \text{for } i \in A \\ E\left[\pi_i\right] & \text{for } i \in N \backslash A \end{cases}.$$

The class structure Ω^A is stable if, given the announcements of the other plaintiffs, no class member could increase its expected payoff by opting out of the class and no opt-out plaintiff could increase its expected payoff by remaining in the class. Formally, for a class member $i \in A$, let Ω^A_{-i} denote the alternative class structure in which plaintiff i opts out of the class action, i.e., $\Omega^A_{-i} = \left\{A \backslash \{i\}, i, (j)_{j \in N \backslash A}\right\}$. I refer to $v_i(\Omega^A_{-i})$ as class member i's *outside option*. Similarly, for an opt-out plaintiff $i \in N \backslash A$, let Ω^A_{+i} denote the alternative class structure in which plaintiff i remains the class action, i.e., $\Omega^A_{+i} = \left\{A \cup \{i\}, (j)_{j \in N \backslash A \cup \{i\}}\right\}$. I refer to $v_i(\Omega^A_{+i})$ as opt-out plaintiff i's *inside option*. The class structure Ω^A is *internally stable* if for each class member $i \in A$ its expected payoff under Ω^A is greater than its outside option:

$$v_i(\Omega^A) \geq v_i(\Omega^A_{-i}) \text{ for all } i \in A. \tag{2.18}$$

The class structure Ω^A is *externally stable* if for each opt-out plaintiff $i \in N \backslash A$ its expected payoff under Ω^A is greater than or equal to its inside option:

$$v_i(\Omega^A) \geq v_i(\Omega^A_{+i}) \text{ for all } i \in N \backslash A. \tag{2.19}$$

The class structure Ω^A is *stable* if it is both internally stable and externally stable. Note that this notion of stability corresponds to the concept of pure strategy Nash equilibrium: the class structure Ω^A is stable if and only if the announcement profile that induces Ω^A constitutes a pure strategy Nash equilibrium of game Γ.[24]

For purposes of this chapter, I focus on the stability of the class structure Ω^N, which consists of the class of all plaintiffs, $A = N$, and no opt-out plaintiffs. I refer to Ω^N as the *global class.* Note that the global class is stable provided it is internally stable; it is trivially externally stable because there are no opt-out plaintiffs. I focus on the global class for two reasons. First, it is the default class structure. The global class is formed by operation of law upon certification of a class action under Rule 23. Second, it presumably is the efficient class structure. Class certification under Rule 23(b)(3) on behalf of all plaintiffs implies that "a class action is superior to other available methods for fairly and efficiently adjudicating the controversy" (Fed. R. Civ. P. 23(b)(3)) and that it is inappropriate to divide the global class into subclasses (Fed. R. Civ. P. 23(c)(4)).

In particular, I examine the *asymptotic stability* of the global class.

DEFINITION 2.1 *The global class is* asymptotically stable *if and only if for every plaintiff* $i \in N$, $\operatorname*{plim}_{n\to\infty} \left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0.$

According to Definition 2.1, the global class is asymptotically stable if and only if the probability that it is (internally) stable converges to one as the number of

[24] This notion of stability was introduced by d'Aspremont et al. (1983). My formulation closely follows Weikard (2009).

plaintiffs becomes arbitrarily large.[25] As noted above, I examine the asymptotic stability of the global class because the situation under consideration is a *mass* tort class action in which the class is "numerous" (Fed. R. Civ. P. 23(a)(1)).

2.3 ASYMPTOTIC STABILITY OF THE GLOBAL CLASS

This section examines the asymptotic stability of the global class under each allocation rule. I show that the global class is asymptotically stable if the net recovery of the class is allocated pro rata in accordance with the members' outside options ($\mathcal{R}^3$), but that the global class is not necessarily asymptotically stable if the net recovery of the class is shared equally by the members ($\mathcal{R}^1$) or allocated pro rata in accordance with their damage claims ($\mathcal{R}^2$). For $\mathcal{R}^1$ and $\mathcal{R}^2$, I derive necessary and sufficient conditions for the asymptotic stability of the global class as well as sufficient conditions for the asymptotic stability and instability of the global class. In addition, I show that the asymptotic stability of the global class under $\mathcal{R}^1$ necessarily implies the asymptotic stability of the global class under and $\mathcal{R}^2$ but not vice versa.

Before proceeding with the analysis by allocation rule, I note the following prefatory results, which hold for every allocation rule.

LEMMA 2.1 *Take any allocation rule.*

(a) *Take any $A \subset N$. Then for all $i \in N \backslash A$, $\phi_i \leq \phi_A$ if and only if*

$$\theta_i > \left(\frac{C+K}{\frac{1}{|A|}(C_A+K_A)}\right)\left(\frac{1}{|A|}\theta_A\right).$$

(b) *Define $\theta_{(1)} = \min_{1 \leq i \leq n} \theta_i$ and $\phi_{(1)} = F_{\Delta(W)}\left(\frac{C+K}{\theta_{(1)}}\right)$. Then $\phi_{(1)} \geq \phi_N$.*

(c) *Define $\theta_{(n)} = \max_{1 \leq i \leq n} \theta_i$ and $\phi_{(n)} = F_{\Delta(W)}\left(\frac{C+K}{\theta_{(n)}}\right)$. Then:*

[25] To see this, note that $\operatorname*{plim}_{n\to\infty}\left(v_i(\Omega^A) - v_i(\Omega^A_{-i})\right) = d \geq 0$ if and only if for all $e > 0$, $\lim_{n\to\infty} \Pr(\left|\left(v_i(\Omega^A) - v_i(\Omega^A_{-i})\right) - d\right| < e) = 1$.

(i) $\operatorname*{plim}_{n\to\infty} \theta_{(n)} = \overline{\theta}$*;*

(ii) $\operatorname*{plim}_{n\to\infty} \left(\phi_{(n)} - \phi_N\right) \leq 0$ *if* $\overline{\theta} > \left(\frac{C+K}{c+k}\right) E[\theta]$*; and*

(iii) $\operatorname*{plim}_{n\to\infty} \left(\phi_{(n)} - \phi_N\right) \geq 0$ *if* $\overline{\theta} < \left(\frac{C+K}{c+k}\right) E[\theta]$*.*

PROOF. (a) Follows immediately from the fact that $F_{\Delta(W)}$ is nondecreasing.

(b) By assumptions (2.13)-(2.15) and because f_θ is strictly positive on $[\underline{\theta}, \overline{\theta}]$, we have $\frac{C+K}{\frac{1}{n}(C_N+K_N)} \geq 1 > \frac{\theta_{(1)}}{\frac{1}{n}\theta_N}$. Because $F_{\Delta(W)}$ is nondecreasing, this implies $\phi_{(1)} = F_{\Delta(W)}\left(\frac{C+K}{\theta_{(1)}}\right) \geq F_{\Delta(W)}\left(\frac{C_N+K_N}{\theta_N}\right) = \phi_N$.

(c) (i) For any $\varepsilon > 0$, $\Pr\left(\left|\theta_{(n)} - \overline{\theta}\right| \geq \varepsilon\right) = \Pr\left(\theta_{(n)} \geq \overline{\theta} + \varepsilon\right) + \Pr\left(\theta_{(n)} \leq \overline{\theta} - \varepsilon\right)$. Note that $\Pr\left(\theta_{(n)} \geq \overline{\theta} + \varepsilon\right) = 0$ (because $\theta_i \leq \overline{\theta}$ for all $i \in N$). Note further that $\Pr\left(\theta_{(n)} \leq \overline{\theta} - \varepsilon\right) = \left[F_\theta(\overline{\theta} - \varepsilon)\right]^n$ (see, e.g., Casella and Berger 2002, thm. 5.4.4) and that $F_\theta(\overline{\theta} - \varepsilon) \in [0, 1)$ (because $\overline{\theta}$ is the upper bound of the support set of F_θ). It follows that $\lim_{n\to\infty} \Pr\left(\left|\theta_{(n)} - \overline{\theta}\right| \geq \varepsilon\right) = \lim_{n\to\infty}\left[F_\theta(\overline{\theta} - e)\right]^n = 0$.

(ii)-(iii) Because $F_{\Delta(W)}$ is continuous, $\operatorname{plim}_{n\to\infty} \phi_{(n)} = F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)$ and $\operatorname{plim}_{n\to\infty} \phi_N = F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)$ by the continuous mapping theorem (see, e.g., Casella and Berger 2002, thm. 5.5.4). Because $F_{\Delta(W)}$ is nondecreasing, $\overline{\theta} > \left(\frac{C+K}{c+k}\right) E[\theta]$ implies $\operatorname{plim}_{n\to\infty}\left(\phi_{(n)} - \phi_N\right) = \operatorname{plim}_{n\to\infty} \phi_{(n)} - \operatorname{plim}_{n\to\infty} \phi_N \leq 0$ and $\overline{\theta} < \left(\frac{C+K}{c+k}\right) E[\theta]$ implies $\operatorname{plim}_{n\to\infty}\left(\phi_{(n)} - \phi_N\right) = \operatorname{plim}_{n\to\infty} \phi_{(n)} - \operatorname{plim}_{n\to\infty} \phi_N \geq 0$. ■

Lemma 2.1(a) says that an opt-out plaintiff is less likely to settle, and therefore more likely to litigate, than the class if and only if its damage claim exceeds the average damage claim of the class members by a factor greater than the scale benefit of the class action. Lemma 2.1(b) says that the member of the global class with the lowest damage claim would be more likely to settle, and therefore less likely to litigate, than the global class were that member to opt out. Lemma 2.1(c)(i) says that, as the number of plaintiffs becomes arbitrarily large, the probability that at least one plaintiff has the maximum damage claim converges to one. Lemmas 2.1(c)(ii) and (iii) say that, as the number of plaintiffs becomes arbitrarily large, the probability that the member of the global class with the highest damage claim would be less (more) likely to settle, and therefore more (less) likely to litigate, than the global class were that member to opt out converges to one, provided that the factor by which the maximum damage claim exceeds the expected damage claim is greater (less) than the maximum scale benefit of a class action.

2.3.1 Equal sharing ($\mathcal{R}^1$)

The following proposition sets forth a necessary and sufficient condition for the asymptotic stability of the global class under equal sharing.

PROPOSITION 2.1 *Suppose the allocation rule is* $\mathcal{R}^1$. *Then the global class is asymptotically stable if and only if*

$$\overline{\theta} \leq E[\theta] + \xi_1 \tag{2.20}$$

where $\xi_1 = \frac{1}{W}\left((C-c) + \lambda\left[F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\right)$.

PROOF. Under $\mathcal{R}^1$, for all $i \in N$,

$$\begin{aligned} v_i(\Omega^N) &= \frac{1}{n}E[\pi_N] \\ &= \phi_N\lambda\left(E[Q_N]\frac{\theta_N}{n} + \frac{K_N}{n}\right) + (1-\phi_N\lambda)\left(E[P_N]\frac{\theta_N}{n} - \frac{C_N}{n}\right) \\ &= \left(W\frac{\theta_N}{n} - \frac{C_N}{n}\right) + \phi_N\lambda\left(\frac{C_N+K_N}{n}\right). \end{aligned}$$

In addition, for all $i \in N$,

$$\begin{aligned} v_i(\Omega^N_{-i}) &= E[\pi_i] \\ &= \phi_i\lambda\left(E[Q_i]\theta_i + K\right) + (1-\phi_i\lambda)\left(E[P_i]\theta_i - C_i\right) \\ &= (W\theta_i - C) + \phi_i\lambda(C+K). \end{aligned}$$

Note $\text{plim}_{n\to\infty}\left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0 \Leftrightarrow \text{plim}_{n\to\infty} v_i(\Omega^N) - \text{plim}_{n\to\infty} v_i(\Omega^N_{-i}) \geq 0$. Without loss of generality, label the plaintiff with the highest damage claim as plaintiff (n). That is, $\theta_{(n)} = \max_{1\leq i\leq n}\theta_i$. Because $\theta_{(n)} \geq \theta_i$ and $\phi_{(n)} \leq \phi_i$ for all $i \in N$, we have $\text{plim}_{n\to\infty} v_{(n)}(\Omega^N_{-(n)}) \geq \text{plim}_{n\to\infty} v_i(\Omega^N_{-i})$ for all $i \in N$. It follows that $\text{plim}_{n\to\infty}\left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0$ for all $i \in N \Leftrightarrow \text{plim}_{n\to\infty} v_{(n)}(\Omega^N) - \text{plim}_{n\to\infty} v_{(n)}(\Omega^N_{-(n)}) \geq 0$.

By assumption, $\frac{\theta_N}{n} = \frac{E[\theta]\cdot n}{n} = E[\theta]$. By assumption (2.15), $\text{plim}_{n\to\infty}\frac{C_N}{n} = c$ and $\text{plim}_{n\to\infty}\frac{C_N+K_N}{n} = c+k$. By Lemma 1(c), $\text{plim}_{n\to\infty}\phi_N = F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)$, $\text{plim}_{n\to\infty}\theta_{(n)} = \overline{\theta}$, and $\text{plim}_{n\to\infty}\phi_{(n)} = F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)$. Thus,

$$\begin{aligned}
\underset{n\to\infty}{\text{plim}}\, v_{(n)}(\Omega^N) &= \underset{n\to\infty}{\text{plim}}\left[\left(W\frac{\theta_N}{n} - \frac{C_N}{n}\right) + \phi_N\lambda\left(\frac{C_N+K_N}{n}\right)\right] \\
&= \left(WE[\theta] - c\right) + \lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)
\end{aligned}$$

and

$$\begin{aligned}
\underset{n\to\infty}{\text{plim}}\, v_{(n)}(\Omega^N_{-(n)}) &= \underset{n\to\infty}{\text{plim}}\left[\left(W\theta_{(n)} - C\right) + \phi_{(n)}\lambda\left(C+K\right)\right] \\
&= \left(W\overline{\theta} - C\right) + \lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K).
\end{aligned}$$

Therefore, $\text{plim}_{n\to\infty}\left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0$ for all $i \in N$

$$\begin{aligned}
\Leftrightarrow &\left(WE[\theta] - c\right) + \lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) \\
&\qquad - \left(W\overline{\theta} - C\right) - \lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K) \geq 0
\end{aligned}$$

$\Leftrightarrow \overline{\theta} \leq E[\theta] + \frac{1}{W}\left((C-c) + \lambda\left[F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\right)$.

■

The following results follow from Proposition 2.1.

Corollary 2.1 *Suppose the allocation rule is* $\mathcal{R}^1$. *Then:*

(a) *The global class is not asymptotically stable if* $E[\theta] < \overline{\theta} - \underline{\theta}$.

(b) *If* $E[\theta] > \overline{\theta} - \underline{\theta}$, *then:*

(i) *the global class is asymptotically stable if* $\frac{C+K}{c+k}$ *is sufficiently high and* λ *is sufficiently low; and*

(ii) *the global class is not asymptotically stable if* $\frac{C+K}{c+k}$ *is sufficiently low and either* W *is sufficiently high or* λ *is sufficiently low.*

Proof. (a) Assume $E[\theta] < \overline{\theta} - \underline{\theta}$. It follows that $\overline{\theta} > E[\theta] + \xi_1$ if $\xi_1 \leq \underline{\theta}$. By definition,

$$\xi_1 = \frac{1}{W}\left((C-c) + \lambda\left[F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\right).$$

Let

$$\Gamma = \lambda\left[F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right].$$

If $\Gamma < 0$, then $\xi_1 \leq \frac{C}{W} \leq \underline{\theta}$ because $W\underline{\theta} - C \geq 0$. If $\Gamma > 0$, then

$$\xi_1 \leq \frac{1}{W}\left((C-c) + \left[(c+k) - \frac{1}{2}(C+K)\right]\right)$$

because $\lambda \in [0,1]$ and $F_{\Delta(W)}(z) \in \left[\frac{1}{2}, 1\right]$ for $z \geq 0$. It follows that

$$\xi_1 \leq \frac{1}{2}\left(\frac{C}{W} + \frac{K}{W}\right) \leq \underline{\theta}$$

because $k \leq K$, $W\underline{\theta} - C \geq 0$, and $W\underline{\theta} - K \geq 0$.

(b) (i) Assume $E[\theta] > \overline{\theta} - \underline{\theta}$. Suppose $\lambda = 0$. Then $\xi_1 = \frac{C-c}{W}$. Recall that $W\underline{\theta} - C \geq 0$. It follows that $\xi_1 \leq \underline{\theta} - \frac{c}{W}$. Because $C + K \leq 2W\underline{\theta}$, $\frac{C+K}{c+k} \to \infty$

implies $c+k \to 0$, which in turn implies $c \to 0$. Therefore, $\xi_1 \to \underline{\theta}$ from below as $\frac{C+K}{c+k} \to \infty$. It follows that there exists $x > 0$ such that $\frac{C+K}{c+k} > x$ implies $E[\theta] + \underline{\theta} > E[\theta] + \xi_1 > \overline{\theta}$. Therefore, by continuity of ξ_1, there exist $\delta_\lambda > 0$ and $x > 0$ such that if $\lambda < \delta_\lambda$ then $\frac{C+K}{c+k} > x$ implies $\overline{\theta} < E[\theta] + \xi_1$.

(ii) Suppose $\frac{C+K}{c+k} = 1$ and $W = 1$. Note that because $0 < c \leq C$ and $0 < k \leq K$, $\frac{C+K}{c+k} = 1$ implies $C = c$. Note further that because $F_{\Delta(1)}\left(\frac{c+k}{E[\theta]}\right) = F_{\Delta(1)}\left(\frac{C+K}{\overline{\theta}}\right) = 1$, $W = 1$ implies $\xi_1 = ((C-c) + \lambda[(c+k) - (C+K)])$. It follows that $\xi_1 = 0$ when $\frac{C+K}{c+k} = 1$ and $W = 1$. Therefore, by continuity of ξ_1, there exist $\delta_C > 1$ and $\delta_W < 1$ such that if $\frac{C+K}{c+k} < \delta_C$ and $W > \delta_W$ then $\xi_1 < \overline{\theta} - E[\theta]$. Now suppose $\frac{C+K}{c+k} = 1$ and $\lambda = 0$. Because $\frac{C+K}{c+k} = 1$ implies $C = c$, it follows that $\xi_1 = 0$. Therefore, by continuity of ξ_1, there exist $\delta_C > 1$ and $\delta_\lambda > 0$ such that if $\frac{C+K}{c+k} < \delta_C$ and $\lambda < \delta_\lambda$ then $\xi_1 < \overline{\theta} - E[\theta]$. ■

Corollary 2.1(a) implies that a key determinant of the asymptotic stability of the global class under $\mathcal{R}^1$ is the shape of the distribution of the plaintiffs' damage claims. It suggests that the global class is more likely to be asymptotically stable under $\mathcal{R}^1$ if the expected damage claim is high and the range of damage claims is narrow. If the distribution of the plaintiffs' damage claims is unimodal, Corollary 2.1(a) implies that the global class is more likely to be asymptotically stable under $\mathcal{R}^1$ if the distribution is negatively skewed. Corollary 2.1(b) suggests that the global class is more likely to be asymptotically stable under $\mathcal{R}^1$ if the scale benefits of a class action are high and the plaintiffs' bargaining power in settlement negotiations is low. If the scale benefits of a class action are low, however, Corollary 2.1(b) suggests that the global class is less likely to be asymptotically stable under $\mathcal{R}^1$ if the plaintiffs' probability of prevailing at trial is high or their bargaining power in settlement negotiations is low.

2.3.2 Pro Rata by Damage Claims ($\mathcal{R}^2$)

The following proposition sets forth a necessary and sufficient condition for the asymptotic stability of the global class if the net recovery of the class is allocated pro rata in accordance with the members' damage claims.

PROPOSITION 2.2 *Suppose the allocation rule is $\mathcal{R}^2$. Then the global class is asymptotically stable if and only if*

$$\overline{\theta} \leq \frac{C}{c} E[\theta] + \xi_2 \tag{2.21}$$

where $\xi_2 = \frac{\lambda}{c}\left[\overline{\theta} F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - E[\theta]\, F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]$.

PROOF. Under $\mathcal{R}^2$, for all $i \in N$,

$$\begin{aligned} v_i(\Omega^N) &= \frac{\theta_i}{\theta_N} E[\pi_N] = \frac{\theta_i}{\frac{1}{n}\theta_N} \frac{1}{n} E[\pi_N] \\ &= \frac{\theta_i}{\frac{1}{n}\theta_N}\left[\left(W\frac{\theta_N}{n} - \frac{C_N}{n}\right) + \phi_N \lambda \left(\frac{C_N + K_N}{n}\right)\right]. \end{aligned}$$

Without loss of generality, label the plaintiff with the highest damage claim as plaintiff (n). By the same logic in the proof of Proposition 2.1, it follows that

$$\operatorname*{plim}_{n\to\infty}\left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0 \text{ for all } i \in N \Leftrightarrow \operatorname*{plim}_{n\to\infty} v_{(n)}(\Omega^N) - \operatorname*{plim}_{n\to\infty} v_{(n)}(\Omega^N_{-(n)}) \geq 0.$$

Now

$$\begin{aligned} \operatorname*{plim}_{n\to\infty} v_{(n)}(\Omega^N) &= \operatorname*{plim}_{n\to\infty}\left(\frac{\theta_{(n)}}{\frac{1}{n}\theta_N}\left[\left(W\frac{\theta_N}{n} - \frac{C_N}{n}\right) + \phi_N\lambda\left(\frac{C_N+K_N}{n}\right)\right]\right). \\ &= \frac{\overline{\theta}}{E[\theta]}\left[(WE[\theta] - c) + \lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)\right]. \end{aligned}$$

Therefore, $\operatorname{plim}_{n\to\infty}\left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0$ for all $i \in N$

$$\Leftrightarrow \frac{\overline{\theta}}{E[\theta]}\left[(WE[\theta]-c)+\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)\right] - \left(W\overline{\theta}-C\right) - \lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K) \geq 0$$

$$\Leftrightarrow WE[\theta]\,\overline{\theta} - c\overline{\theta} + \lambda\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - WE[\theta]\,\overline{\theta} + CE[\theta] - \lambda E[\theta]\,F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K) \geq 0$$

$$\Leftrightarrow \overline{\theta} \leq \tfrac{C}{c}E[\theta] + \tfrac{\lambda}{c}\left[\overline{\theta}F_{\Delta(W)}\left(\tfrac{c+k}{E[\theta]}\right)(c+k) - E[\theta]\,F_{\Delta(W)}\left(\tfrac{C+K}{\overline{\theta}}\right)(C+K)\right]. \quad \blacksquare$$

The following results follow from Proposition 2.2.

Corollary 2.2 *Suppose the allocation rule is $\mathcal{R}^2$. Then:*

(a) *The global class is asymptotically stable if $\frac{C+K}{c+k}$ is sufficiently high and λ is sufficiently low; in particular, if*
$\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}$ *and* $\lambda \leq \frac{c\overline{\theta}-CE[\theta]}{\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)-E[\theta]F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)}$.

(b) *The global class is asymptotically stable if*
$\left(\frac{\overline{\theta}}{E[\theta]} - \frac{K}{c}\right)\left(1+\frac{k}{c}\right)^{-1} \leq \frac{C+K}{c+k} \leq \frac{\overline{\theta}}{E[\theta]}$.

(c) *The global class is not asymptotically stable if $\frac{C+K}{c+k}$ is sufficiently low and either (i) W is sufficiently high and $\lambda < C$ or (ii) λ is sufficiently low.*

Proof. (a) Rewrite condition (2.21) as $\xi_2 \geq \overline{\theta} - \frac{C}{c}E[\theta]$. This holds if

$$\lambda\left[\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - E[\theta]\,F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right] \geq c\overline{\theta} - CE[\theta].$$

Now if $\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}$, then $F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right) < F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)$ because $F_{\Delta(W)}$ is non-decreasing. It follows that $\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}\frac{F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)}{F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)}$, or $\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) < E\left[\theta\right]F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)$. In addition, $\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}$ implies $c\overline{\theta} < CE\left[\theta\right]$. To see this, let $\frac{C+K}{c+k} = x$. Then we have $\frac{C}{c} = \left(1+\frac{k}{c}\right)x - \frac{K}{c} < x < \frac{\overline{\theta}}{E[\theta]}$. It follows that condition (2.21) holds if $\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}$ and $\lambda \leq \frac{c\overline{\theta}-CE[\theta]}{\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)-E[\theta]F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)}$.

(b) Suppose $\frac{C+K}{c+k} \leq \frac{\overline{\theta}}{E[\theta]}$. Then $F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right) > F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)$. It follows that $\frac{C+K}{c+k} \leq \frac{\overline{\theta}}{E[\theta]}\frac{F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)}{F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)}$, or $\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) \geq E\left[\theta\right]F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)$. Thus, $\frac{C+K}{c+k} \leq \frac{\overline{\theta}}{E[\theta]}$ implies $\xi_2 \geq 0$. It follows that if $\frac{C+K}{c+k} \leq \frac{\overline{\theta}}{E[\theta]}$, then condition (2.21) holds if $\overline{\theta} \leq \frac{C}{c}E\left[\theta\right]$. Now let $\frac{C+K}{c+k} = x$. Then $\frac{C}{c} = \left(1+\frac{k}{c}\right)x - \frac{K}{c}$ and we can rewrite the foregoing condition as $\overline{\theta} \leq \left(1+\frac{k}{c}\right)x - \frac{K}{c}E\left[\theta\right]$. This holds if $x \geq \left(\frac{\overline{\theta}}{E[\theta]} - \frac{K}{c}\right)\left(1+\frac{k}{c}\right)^{-1}$. Therefore, condition (2.21) holds if $\left(\frac{\overline{\theta}}{E[\theta]} - \frac{K}{c}\right)\left(1+\frac{k}{c}\right)^{-1} \leq \frac{C+K}{c+k} \leq \frac{\overline{\theta}}{E[\theta]}$.

(c) (i) Suppose $\frac{C+K}{c+k} = 1$, $W = 1$, and $C > \lambda$. Let $g = \frac{C}{c}E\left[\theta\right] + \xi_2$. We know from the proof of Corollary 2.1(b)(ii) that $\frac{C+K}{c+k} = 1$ implies $C = c$ and that $W = 1$ implies $\xi_2 = \frac{\lambda}{c}\left[\overline{\theta}\left(c+k\right) - E\left[\theta\right](C+K)\right]$. It follows that $g = \frac{\lambda}{C}\overline{\theta} - \left(1-\frac{\lambda}{C}\right)E\left[\theta\right] < \overline{\theta}$ when $\frac{C+K}{c+k} = 1$, $W = 1$, and $C > \lambda$. Therefore, by continuity of g, there exist $\delta_C > 1$ and $\delta_W < 1$ such that if $\frac{C+K}{c+k} < \delta_C$ and $W > \delta_W$ and $C > \lambda$ then $g < \overline{\theta}$.

(ii) Let $g = \frac{C}{c}E\left[\theta\right] + \xi_2$. From part (i) above we know that $C = c$ when $\frac{C+K}{c+k} = 1$. It follows that $g = E\left[\theta\right] < \overline{\theta}$ when $\frac{C+K}{c+k} = 1$ and $\lambda = 0$. Therefore, by continuity of g, there exist $\delta_C > 1$ and $\delta_\lambda > 0$ such that if $\frac{C+K}{c+k} < \delta_C$ and $\lambda < \delta_\lambda$ then $g < \overline{\theta}$. ∎

The results of Corollaries 2.2(a) and (c) closely resemble those of Corollaries 2.1(b)(i) and (ii). They suggest that if the scale benefits of a class action are high, the global class is more likely to be asymptotically stable under $\mathcal{R}^2$ if the plaintiffs' bargaining power in settlement negotiations is low, and that if the scale benefits of a class action are low, the global class is less likely to be asymptotically stable under $\mathcal{R}^2$ if the plaintiffs' probability of prevailing at trial is high or their bargaining power in settlement negotiations is low. Corollary 2.2(b) suggests that, irrespective of the plaintiffs' probability of prevailing at trial or bargaining power in settlement negotiations, the global class is likely to be asymptotically stable under $\mathcal{R}^2$ if the maximum scale benefits of a class action are close to (but do not exceed) the ratio of the maximum damage claim to the expected damage claim.

It is interesting to note how the results of Lemma 2.1 inform certain results of Corollaries 2.1 and 2.2. First, Corollaries 2.1(b)(i) and 2.2(a) indicate that even if the scale benefits of a class action are high, damage averaging may lead the global class to unravel if the plaintiffs' bargaining power in settlement negotiations is high. Lemma 2.1(c) suggests why: when the scale benefits of a class action are high, the probability of reaching a settlement (and realizing the benefits of their high bargaining power) is greater for opt-plaintiffs than it is for the global class. Second, Corollaries 2.1(b)(ii) and 2.2(c) indicate that when the scale benefits of a class action are low, damage averaging may lead the global class to unravel if the plaintiffs' probability of prevailing at trial is high. Again Lemma 2.1(c) suggests why: when the scale benefits of a class action are low, the probability of litigation (and realizing the benefits of their high probability of prevailing at trial) is greater for opt-out plaintiffs than it is for the global class.

The following proposition states that asymptotic stability of the global class under $\mathcal{R}^1$ necessarily implies asymptotic stability of the global class under $\mathcal{R}^2$ but not vice versa.

PROPOSITION 2.3 *If the global class is asymptotically stable under $\mathcal{R}^1$, then the global class is asymptotically stable under $\mathcal{R}^2$. If the global class is asymptotically stable under $\mathcal{R}^2$, however, the global class may or may not be asymptotically stable under $\mathcal{R}^1$.*

Proof. Assume $\overline{\theta} \leq E[\theta] + \xi_1$. This implies $\xi_1 > 0$ because $E[\theta] < \overline{\theta}$. It follows that

$$\frac{1}{W}\left((C-c) + \lambda\left[F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\right) > 0,$$

which implies

$$\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) > \lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K) - (C-c).$$

Recall that $W\overline{\theta} - C > 0$, $\underline{\theta} < E[\theta] < \overline{\theta}$, and $0 < c \leq C$. This implies $W\overline{\theta} - c > WE[\theta] - c > 0$. In addition, note that $\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) > 0$. It follows that

$$\begin{aligned}\left(\frac{W\overline{\theta}-c}{c}\right)&\left[\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)\right]\\ &> \left(\frac{WE[\theta]-c}{c}\right)\left[\lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K) - (C-c)\right],\end{aligned}$$

which implies

$$\begin{aligned}\left(\frac{WE[\theta]-c}{c}\right)(C-c) > &\left(\frac{WE[\theta]-c}{c}\right)\left[\lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\\ &- \left(\frac{W\overline{\theta}-c}{c}\right)\left[\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)\right].\end{aligned}$$

It follows that

$$\begin{aligned}\left(\frac{C-c}{c}\right)E[\theta] > \left(\frac{C-c}{W}\right) + &\left(\frac{E[\theta]}{c} - \frac{1}{W}\right)\left[\lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\\ &- \left(\frac{\overline{\theta}}{c} - \frac{1}{W}\right)\left[\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)\right],\end{aligned}$$

which implies

$$
\begin{aligned}
&\frac{C}{c}E[\theta] - E[\theta] \\
&\quad > \frac{1}{W}\left((C-c) + \lambda\left[F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right]\right) \\
&\qquad\qquad - \frac{\lambda}{c}\left[\overline{\theta}F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k) - E[\theta]\,F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right],
\end{aligned}
$$

or

$$\frac{C}{c}E[\theta] - E[\theta] > \xi_1 - \xi_2.$$

Hence, $E[\theta] + \xi_1 < \frac{C}{c}E[\theta] + \xi_2$. Thus, $\overline{\theta} \leq E[\theta] + \xi_1$ implies $\overline{\theta} < \frac{C}{c}E[\theta] + \xi_2$. ■

2.3.3 Pro Rata by Outside Options ($\mathcal{R}^3$)

The following proposition states that the global class is asymptotically stable if the net recovery of the class is allocated to the members pro rata in accordance with their outside options.

PROPOSITION 2.4 *Suppose the allocation rule is* $\mathcal{R}^3$. *Then the global class is asymptotically stable.*

PROOF. Under $\mathcal{R}^3$, for all $i \in N$,

$$v_i(\Omega^N) = \frac{E[\pi_i]}{\sum_{j=1}^{n} E[\pi_j]} E[\pi_N] = \frac{E[\pi_i]}{\sum_{j=1}^{n} \frac{1}{n} E[\pi_j]} \frac{1}{n} E[\pi_N].$$

Without loss of generality, label the plaintiff with the highest damage claim as plaintiff (n). By the same logic in the proof of Proposition 2.1, it follows that

$$\operatorname*{plim}_{n\to\infty}\left(v_i(\Omega^N) - v_i(\Omega^N_{-i})\right) \geq 0 \text{ for all } i \in N \Leftrightarrow \operatorname*{plim}_{n\to\infty}\left(v_{(n)}(\Omega^N) - v_{(n)}(\Omega^N_{-(n)})\right) \geq 0.$$

Now

$$\begin{aligned}
&\operatorname*{plim}_{n\to\infty}\left(v_{(n)}(\Omega^N)-v_{(n)}\left(\Omega^N_{-(n)}\right)\right)=\operatorname*{plim}_{n\to\infty}\left(\frac{E\left[\pi_{(n)}\right]}{\sum_{j=1}^n\frac{1}{n}E\left[\pi_j\right]}\frac{1}{n}E\left[\pi_N\right]-E\left[\pi_{(n)}\right]\right)\\
=&\operatorname*{plim}_{n\to\infty}\left(\left(\frac{\frac{1}{n}E\left[\pi_N\right]}{\sum_{j=1}^n\frac{1}{n}E\left[\pi_j\right]}-1\right)E\left[\pi_{(n)}\right]\right)\\
=&\operatorname*{plim}_{n\to\infty}\left(\begin{array}{c}\left(\frac{W\frac{\theta_N}{n}-\frac{C_N}{n}+\phi_N\lambda\left(\frac{C_N+K_N}{n}\right)}{W\frac{\theta_N}{n}-C+\lambda\left(\frac{1}{n}\sum_{j=1}^n\phi_j\right)(C+K)}-1\right)\\ \cdot\left[W\theta_{(n)}-C+\phi_{(n)}\lambda(C+K)\right]\end{array}\right)\\
=&\left(\frac{WE\left[\theta\right]-c+\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)}{WE\left[\theta\right]-C+\lambda\left(\operatorname{plim}_{n\to\infty}\left(\frac{1}{n}\sum_{j=1}^n\phi_j\right)\right)(C+K)}-1\right)\\
&\cdot\left[W\overline{\theta}-C+\lambda F_{\Delta(W)}\left(\frac{C+K}{\overline{\theta}}\right)(C+K)\right].
\end{aligned}$$

Note that

$$\begin{aligned}
\operatorname*{plim}_{n\to\infty}\left(\frac{1}{n}\sum_{j=1}^n\phi_j\right) &= \operatorname*{plim}_{n\to\infty}\left(\frac{1}{n}\right)\left[\operatorname*{plim}_{n\to\infty}\left(\sum_{j=1}^n\phi_j\right)\right]\\
&= \operatorname*{plim}_{n\to\infty}\left(\frac{1}{n}\right)\int_{\frac{C+K}{\overline{\theta}}}^{\frac{C+K}{\underline{\theta}}}F_{\Delta(W)}\left(x\right)dx=0
\end{aligned}$$

because $\int_{\frac{C+K}{\overline{\theta}}}^{\frac{C+K}{\underline{\theta}}}F_{\Delta(W)}\left(x\right)dx\leq\int_{\frac{C+K}{\overline{\theta}}}^{\frac{C+K}{\underline{\theta}}}dx=\frac{C+K}{\underline{\theta}}-\frac{C+K}{\overline{\theta}}<\infty$. In addition, note that

$$\left(WE\left[\theta\right]-c\right)+\lambda F_{\Delta(W)}\left(\frac{c+k}{E\left[\theta\right]}\right)(c+k)\geq WE\left[\theta\right]-C$$

because $C\geq c>0$ and $\lambda F_{\Delta(W)}\left(\frac{c+k}{E[\theta]}\right)(c+k)\geq 0$. Lastly, note that $W\overline{\theta}-C>0$ because $W\underline{\theta}-C\geq 0$ and $\underline{\theta}<\overline{\theta}$. Thus, $\operatorname{plim}_{n\to\infty}\left(v_{(n)}(\Omega^N)-v_{(n)}(\Omega^N_{-(n)})\right)\geq 0$. ■

2.4 MASS TORT SIMULATIONS

In an effort to understand the relative stability of the global class under the three allocation rules, I simulate the model using standard Monte Carlo methods. For pur-

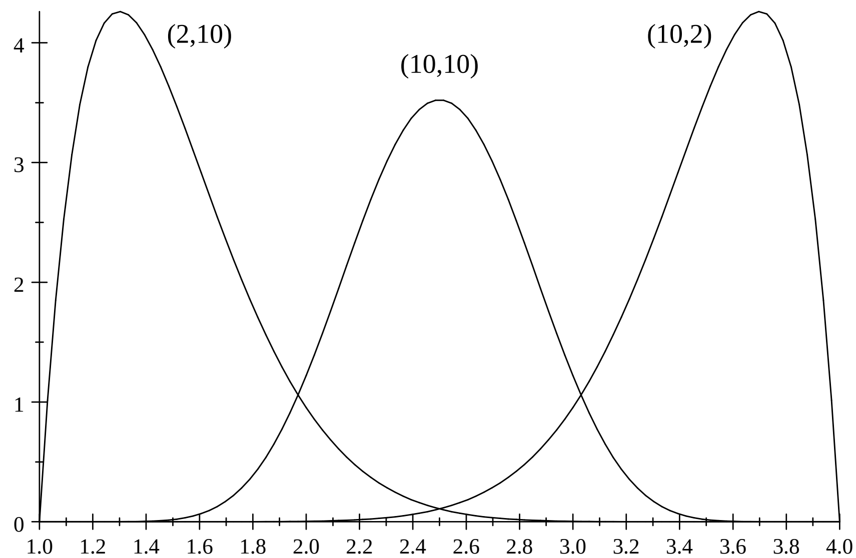


Figure 2.2: Three Densities of θ_i on $[1, 4]$

poses of the simulations, I assume that $\theta_i = \left(\overline{\theta} - \underline{\theta}\right) X_i + \underline{\theta}$, where $X_i \overset{iid}{\sim} Beta(\alpha, \beta)$. That is, I assume that θ_i follows a Beta distribution on the interval $\left[\underline{\theta}, \overline{\theta}\right]$ with shape parameters α and β. Figure 2.2 illustrates three densities of θ_i on $[1, 4]$ for different shape parameters (α, β).

To simulate each mass tort, I follow eight steps/assumptions:

1. $\underline{\theta} = 1,000,000[min(x, y)]$ and $\overline{\theta} = 1,000,000[max(x, y)]$, where x and y are drawn from Uniform$(0, 7)$. I assume that the maximum possible value of $\overline{\theta}$ is \$7 million because it is the median estimated value of a statistical life in the literature (Viscusi and Aldy 2003).

2. Draw α and β from Uniform$(0, 20)$.

3. $E[\theta] = (\overline{\theta} - \underline{\theta}) \left(\frac{\alpha}{\alpha+\beta}\right) + \underline{\theta}$.

4. $C = \frac{1}{3}E[\theta]$. This reflects the standard contingency fee (Eisenberg and Miller 2004a, p. 35).

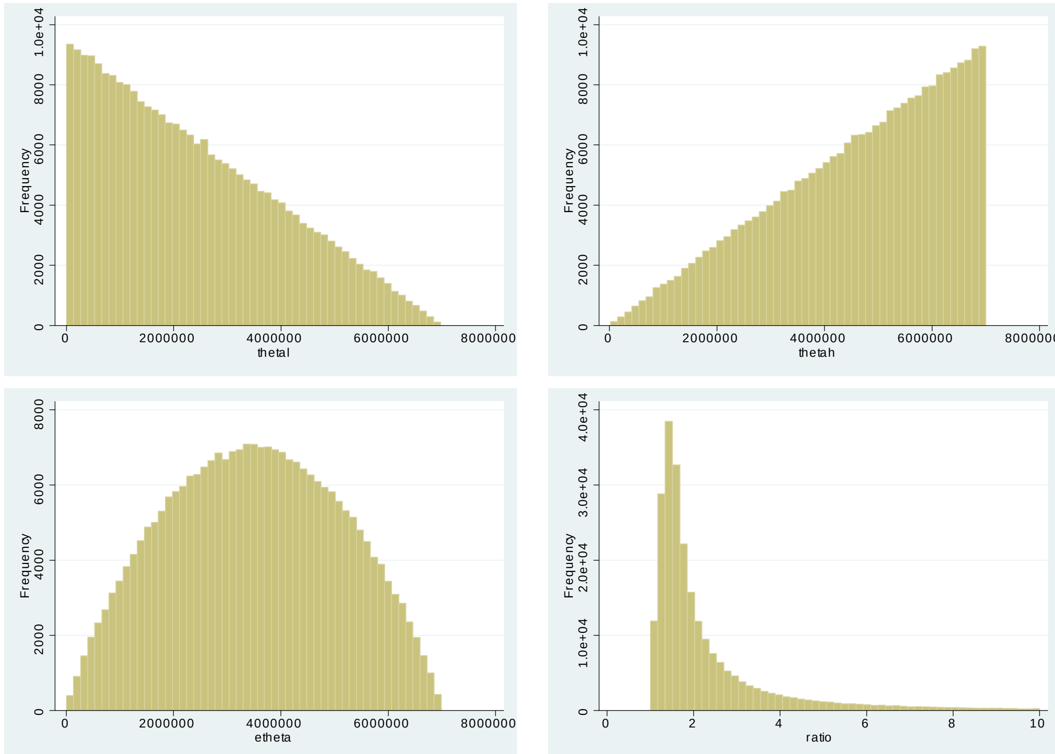


Figure 2.3: Selected Histograms (Raw Data)

5. $c = tC$, where t is drawn from Uniform$(0, 1)$.

6. $k = 2zc$, where z is drawn from Beta$(10, 10)$.

7. $K = (1 + \tau)k$, where τ is drawn from Uniform$(0, 1)$.

8. Draw W and λ from Uniform$(0, 1)$.

I repeat steps 1-8 250,000 times to generate the raw data. Figure 2.3 contains histograms for four variables: $\underline{\theta}$ (thetal), $\overline{\theta}$ (thetah) $E[\theta]$ (etheta), and $\frac{C+K}{c+k}$ (ratio).

To generate the dataset for the stability analysis, I keep only those observations where $W\underline{\theta} - C \geq 0$ and $W\underline{\theta} - K \geq 0$. This leaves me with 93,980 observations. Table 2.1 provides descriptive statistics for the dataset.

Analyzing the dataset for class stability, I find that the frequency with which the global class is asymptotically stable is 0.276 under $\mathcal{R}^1$ (equal sharing) and 0.693

Table 2.1: Descriptive Statistics (Dataset)

Variable	Obs	Mean	Std Dev	Min	Max
$\underline{\theta}$	$93,980$	$3,063,788$	$1,544,757$	$12,534$	$6,973,400$
$\overline{\theta}$	$93,980$	$4,655,124$	$1,654,862$	$21,595$	$6,999,908$
α	$93,980$	9.206	5.894	0.000	20.000
β	$93,980$	10.629	5.682	0.000	20.000
$E[\theta]$	$93,980$	$3,728,501$	$1,600,845$	$17,566$	$6,986,195$
W	$93,980$	0.729	0.172	0.334	1.000
λ	$93,980$	0.499	0.289	0.000	1.000
C	$93,980$	$1,242,834$	$533,615$	$5,855$	$2,328,732$
K	$93,980$	$832,080$	$694,572$	7	$5,684,206$
c	$93,980$	$581,086$	$454,446$	7	$2,311,018$
k	$93,980$	$566,977$	$462,238$	7	$3,241,751$

under $\mathcal{R}^2$ (pro rata by damage claims). In addition, I find that the frequency with which the global class is asymptotically stable under $\mathcal{R}^1$ conditional on asymptotic stability under $\mathcal{R}^2$ is 0.398 and that the frequency with which the global class is asymptotically stable under $\mathcal{R}^2$ conditional on asymptotic stability under $\mathcal{R}^1$ is 1.00.

Thus, as compared to $\mathcal{R}^3$, the global class is asymptotically stable about two-thirds as often under $\mathcal{R}^2$ and about a quarter as often under $\mathcal{R}^1$. Moreover, when the global class is asymptotically stable under $\mathcal{R}^1$, it is always asymptotically stable under $\mathcal{R}^2$ as well, but when the global class is asymptotically stable under $\mathcal{R}^2$, it is asymptotically stable under $\mathcal{R}^1$ about two-fifths of the time. Loosely speaking, therefore, the results suggest that $\mathcal{R}^1$ is about two-fifths as stable as $\mathcal{R}^2$ which is about two-thirds as stable as $\mathcal{R}^3$.

Analyzing the dataset for the determinants of class stability, I find that the simulations confirm the theoretical results in Section 2.3. Conditional frequency tabulations confirm the sharp results in Corollaries 2.1(a) and 2.2(a) and (b): the global class is never asymptotically stable under $\mathcal{R}^1$ when the expected damage claim is less than the difference between the maximum and minimum damage claims ($E[\theta] < \overline{\theta} - \underline{\theta}$), and the global class is always asymptotically stable under $\mathcal{R}^2$ when the conditions set forth in Corollary 2.2(a) or (b) are satisfied.

Tables 2.2-2.5 and Figure 2.4 generally confirm the qualitative results in Corollaries 2.1(b)(i) and (ii) and 2.2(c). Table 2.2 reports the means of key variables for the raw data, for the dataset, and for the subsets of the dataset in which the global class is asymptotically stable under $\mathcal{R}^1$ and $\mathcal{R}^2$ (which subsets I label ASR1 and ASR2, respectively). It also reports p-values of t-tests comparing the means in ASR1 or ASR2, as the case may be, with those in the dataset. For each of $\mathcal{R}^1$ and $\mathcal{R}^2$, Table 2.3 compares the means of key variables for the subsets of the dataset in which the global class is and is not asymptotically stable thereunder and reports p-values of t-tests comparing these means. Figure 2.4 compares the histograms for four variables—$E[\theta]$ (etheta), λ (lambda), $\overline{\theta} - \underline{\theta}$ (range), and $\frac{C+K}{c+k}$ (ratio)—for the subsets of the dataset in which the global class is and is not asymptotically stable under $\mathcal{R}^1$. Tables 2.4 and 2.5 report estimates from four logit regressions. The dependent variable of the model in the left column of Table 2.4 indicates whether the global class is asymptotically stable under $\mathcal{R}^1$; the dependent variable of the model in the right column indicates whether the global class is asymptotically stable under $\mathcal{R}^2$. The dependent variable of both models in Table 2.5 indicates whether the global class is asymptotically stable under $\mathcal{R}^2$; the difference between the models is that the model in the left column is restricted to observations for which $\frac{C+K}{c+k} < \frac{\overline{\theta}}{E[\theta]}$ and the model in the right column is restricted to observations for which $\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}$.

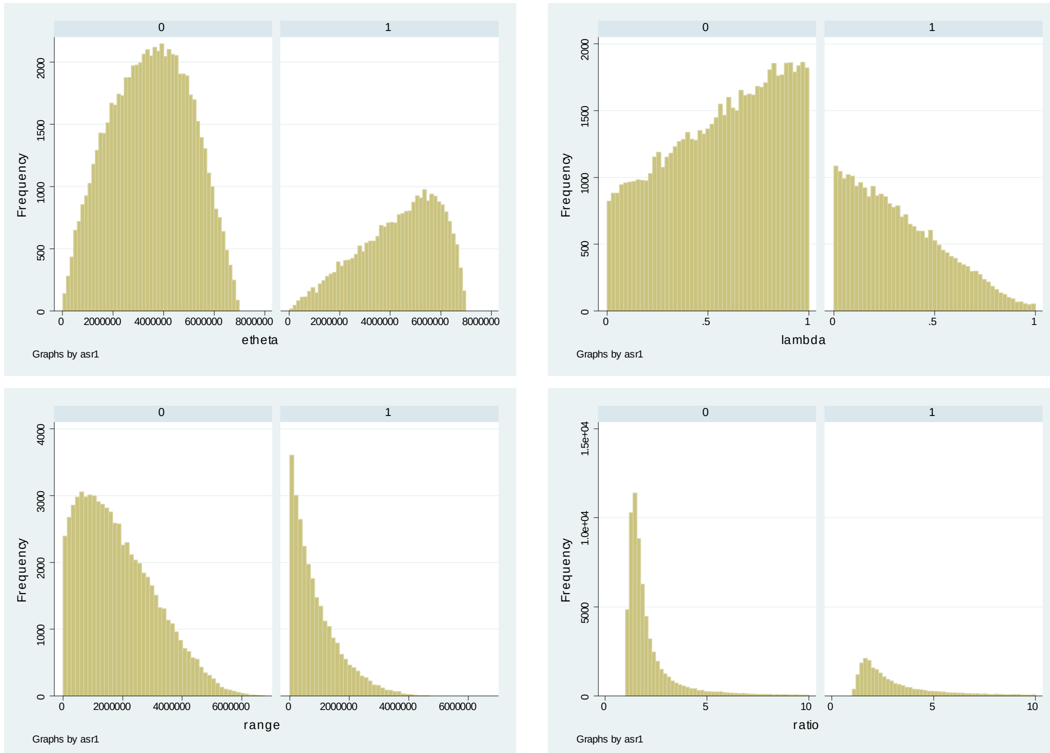
The results on α, β, $E\left[\theta\right]$, and $\overline{\theta} - \underline{\theta}$ in Tables 2.2-2.4 and on $E\left[\theta\right]$ (etheta) and $\overline{\theta} - \underline{\theta}$ (range) in Figure 2.4 show that class stability under $\mathcal{R}^1$ is associated with negatively skewed claims distributions over narrow damages ranges. Likewise, the results on $\frac{C+K}{c+k}$, W, and λ in Tables 2.2-2.4 and on $\frac{C+K}{c+k}$ (ratio) and λ (lambda) in Figure 2.4 show that class stability under $\mathcal{R}^1$ and $\mathcal{R}^2$ is associated with high

Table 2.2: Means Comparisons

	Raw Data	Dataset	R1		R2	
Variable	Mean	Mean	Mean	P	Mean	P
$\underline{\theta}$	$2,330,078$	$3,063,788$	$3,775,801$	0.000	$3,014,127$	0.000
$\overline{\theta}$	$4,663,202$	$4,655,124$	$4,658,449$	0.747	$4,655,194$	0.991
α	9.998	9.206	11.047	0.000	9.149	0.013
β	10.005	10.629	8.709	0.000	10.670	0.064
$E[\theta]$	$3,497,359$	$3,728,501$	$4,329,306$	0.000	$3,696,148$	0.000
W	0.501	0.729	0.676	0.000	0.723	0.000
λ	0.500	0.499	0.322	0.000	0.447	0.000
$\overline{\theta}-\underline{\theta}$	$2,333,124$	$1,591,336$	$882,647$	0.000	$1,641,067$	0.000
$\frac{C+K}{c+k}$	6.522	7.433	13.55	0.013	9.98	0.028
Obs	$250,000$	$93,980$	$25,907$		$65,129$	

Table 2.3: Additional Means Comparisons

	$\mathcal{R}^1$			$\mathcal{R}^2$		
Variable	Not AS	AS	P	Not AS	AS	P
$\underline{\theta}$	$2,792,812$	$3,775,801$	0.000	$3,175,894$	$3,014,127$	0.000
$\overline{\theta}$	$4,653,859$	$4,658,449$	0.704	$4,654,967$	$4,655,194$	0.985
α	8.506	11.047	0.000	9.335	9.149	0.000
β	11.360	8.709	0.000	10.536	10.670	0.001
$E[\theta]$	$3,499,848$	$4,329,306$	0.000	$3,801,535$	$3,696,148$	0.000
W	0.750	0.676	0.000	0.743	0.723	0.000
λ	0.566	0.322	0.000	0.615	0.447	0.000
$\overline{\theta}-\underline{\theta}$	$1,861,047$	$882,647$	0.000	$1,479,073$	$1,641,067$	0.000
$\frac{C+K}{c+k}$	5.11	13.55	0.000	1.69	9.98	0.000
Obs	$68,073$	$25,907$		$28,851$	$65,129$	



Note—For each variable, a Kolmogorov-Smirnov test rejects the equivalence of the distributions.

Figure 2.4: Selected Histograms (Dataset)

Table 2.4: Logit Regressions

	$\mathcal{ASR}^1$			$\mathcal{ASR}^2$		
Variable	Odds Ratio	Robust Std Err	P	Odds Ratio	Robust Std Err	P
α	1.122	0.002	0.000	0.999	0.001	0.474
β	0.868	0.002	0.000	1.000	0.001	0.957
$E[\theta]$ (millions)	1.751	0.015	0.000	0.951	0.005	0.000
$\overline{\theta} - \underline{\theta}$ (millions)	0.165	0.003	0.000	1.154	0.009	0.000
$\frac{C+K}{c+k}$	2.365	0.025	0.000	3.508	0.066	0.000
W (x10)	0.709	0.005	0.000	0.921	0.005	0.000
λ (x10)	0.488	0.002	0.000	0.767	0.002	0.000
Pseudo R^2		0.557			0.218	
Obs		$88,308$			$88,308$	

Note—Regressions exclude observations with $\frac{C+K}{c+k} \geq 10$.

Table 2.5: Additional Logit Regressions

	$AS\mathcal{R}^2\left(\frac{C+K}{c+k} < \frac{\overline{\theta}}{E[\theta]}\right)$			$AS\mathcal{R}^2\left(\frac{C+K}{c+k} > \frac{\overline{\theta}}{E[\theta]}\right)$		
Variable	Odds Ratio	Robust Std Err	P	Odds Ratio	Robust Std Err	P
α	1.024	0.006	0.000	0.988	0.002	0.000
β	0.991	0.004	0.040	1.005	0.002	0.008
$E[\theta]$ (millions)	0.990	0.019	0.616	0.898	0.006	0.000
$\overline{\theta} - \underline{\theta}$ (millions)	1.027	0.018	0.126	1.371	0.015	0.000
$\frac{C+K}{c+k}$	5.132	0.410	0.000	4.409	0.113	0.000
W (x10)	0.869	0.012	0.000	0.904	0.005	0.000
λ (x10)	1.602	0.014	0.000	0.589	0.003	0.000
Pseudo R^2		0.261			0.376	
Obs		$13,991$			$74,317$	

Note—Regressions exclude observations with $\frac{C+K}{c+k} \geq 10$.

scale benefits, low probabilities of plaintiff prevailing at trial, and low plaintiff bargaining power. Finally, the the results on λ in Table 2.5 highlight the nuance that when the scale benefits of a class action are low, class stability under $\mathcal{R}^2$ is associated with high plaintiff bargaining power.

2.5 CONCLUDING REMARKS

This chapter examines the asymptotic stability of the global class in a Rule 23(b)(3) mass tort class action under three rules for allocating the net recovery of the class among its members: (1) equal sharing; (2) pro rata by damage claims; and (3) pro rata by outside options. I analyze a two-stage model of class action formation in which a single defendant faces multiple plaintiffs with heterogeneous damage claims. A global class action is certified at the outset. In stage 1, the formation of the class is modeled as a noncooperative, simultaneous move, single coalition formation game in partition function form. In stage 2, the resolution via litigation or settlement of the class action and any individual actions by opt-out plaintiffs is modeled in the divergent expectations tradition and assumes that if the parties settle their dispute they divide the joint surplus from settlement according to the asymmetric Nash bargaining solution.

I show that the global class is asymptotically stable under rule 3, but may not be asymptotically stable under rules 1 and 2. The shape of the distribution of the plaintiffs' damage claims proves to be a key determinant of class stability under rule 1. In particular, I find that the global class is more likely to be asymptotically stable under rule 1 if the expected damage claim is high and the range of damage claims is narrow, which suggests that if the distribution of the plaintiffs' damage claims is unimodal then the global class is more likely to be asymptotically stable under rule 1 if the distribution is negatively skewed. I also find that the scale benefits of the class action and the plaintiffs' probability of prevailing at trial and their bargaining power in settlement negotiations are important determinants of class stability. Under rules 1 and 2, the global class is more likely to be asymptotically stable when the scale benefits of a class action are high and when the plaintiffs' bargaining power in settlement negotiations is low. When the scale benefits of a class action are low, the global class is less likely to be asymptotically stable under rules 1 and 2 if the plaintiffs' probability of prevailing at trial is high or their bargaining power in settlement negotiations is low.

To supplement the theoretical analysis, I perform Monte Carlo simulations and compare the relative stability of the global class under the three allocation rules. As compared to rule 3, the global class is asymptotically stable about two-thirds as often under rule 2 and about a quarter as often under rule 1. The simulations also confirm my findings regarding the determinants of class stability.

My results highlight a general tradeoff between ex ante and ex post efficiency in selecting an allocation rule in a Rule 23(b)(3) mass tort class action. The tradeoff exists because the governing allocation rule's degree of damage averaging is positively related to the risk that the class will unravel due to adverse selection but negatively related to the cost of implementing the allocation rule. However, the results also provide guidance regarding when this tradeoff may be avoided. Accordingly, they suggest criteria to courts for evaluating whether the predominance and superiority requirements for class certification are satisfied. For example, if the plaintiffs' damage claims are positively skewed over a wide range then perhaps the court should not find that common questions of law or fact predominate over individual questions, or if even a mild degree of damage averaging is likely to destabilize the class then perhaps the court should not find that a class action is superior in terms of efficiency. The results also suggest criteria to attorneys and courts for structuring and approving efficient allocation plans. Given that class actions scholars (and presumably courts) view the standards for judicial review of class action settlements as "confused" and the numerous multifactor tests elaborated by courts as "uncertain in scope, ambiguous in meaning and undefined in weight" (Macey and Miller 2009, pp. 167-168), this would appear to be a welcome contribution.

There are several ways in which this chapter could be extended. First, we could relax the assumption that each opt-out plaintiff must pursue its claim individually. This would require redefining the stability concept from Nash equilibrium to strong Nash equilibrium (Aumann 1959) or coalition-proof Nash equilibrium (Bernheim et al. 1987).[26] Although allowing plaintiffs to form subcoalitions would make the analysis more general,[27] it is not warranted in our setting. The assumption that opt-out plaintiff pursue their claims individually rests on two presumptions, each

[26] Informally, a strategy profile constitutes a strong Nash equilibrium (SNE) if it is immune to deviations by coalitions. A strategy profile constitutes a coalition-proof Nash equilibrium (CPNE) if it is immune to credible deviations by coalitions, i.e., coalitional deviations that themselves are immune to further deviations by subcoalitions. For formal definitions of SNE and CPNE, see, e.g., Bloch (2003).

[27] One consequence of relaxing this assumption would be that Proposition 2.3 would no longer hold; that is, allocating the net recovery of the class to the members pro rata in accordance with their outside options would no longer ensure the asymptotic stability of the global class.

of which is consistent with the premise that a global class action has been certified under Rule 23(b)(3). First, it presumes that no other court would certify a separate class action on behalf of some or all opt-out plaintiffs, which is consistent with the fact that the court did not deem it appropriate to divide the global class into subclasses (Fed. R. Civ. P. 23(c)(4)). Second, it presumes that search costs, personal jurisdiction requirements, or other transaction costs preclude opt-out plaintiffs from maintaining one or more joinder actions under Rule 20, which is consistent with the fact that the court determined that "the class is so numerous that joinder of all members is impracticable" (Fed. R. Civ. P. 23(a)(1)).

Second, we could relax the assumption that there are no externalities or spillovers across plaintiffs. In particular, we could assume that the class action is resolved first and that the existence or size of the class affects the expected recovery of opt-out plaintiffs.[28] For example, we could assume that the class action increases the probability that opt-out plaintiffs would prevail at trial due to the potential for a factual or legal determination in favor of the class to be given preclusive effect against the defendant in a subsequent individual action by an opt-out plaintiff under the doctrine of nonmutual offensive collateral estoppel.[29] We also could relax the assumptions that the defendant's assets are sufficient to satisfy all damage claims and that all plaintiffs have the same priority in bankruptcy. Instead, we could assume that the class action reduces the expected payoff for opt-out plaintiffs due to the potential that, after the resolution of the class action, the defendant

[28] We also could consider making the timing of litigation/settlement endogenous, as in Marceau and Mongrain (2003).

[29] Under this assumption, if the class settles or losses at trial, the probability that an opt-out plaintiff prevails in a subsequent trial is W, but if the class prevails at trial, the probability that an opt-out plaintiff prevails in a subsequent trial is $Y > W$. Accordingly, by the law of total probability, the ex ante probability that an opt-out plaintif would prevail at trial is $W^{+} = [\phi_A + (1-\phi_A)(1-W)]\,W + [(1-\phi_A)\,W]\,Y > W$. The class action, therefore, increases the expected payoff of pursuing individual litigation against the defendant, which serves to undermine the stability of the global class. On the topic of preclusion in class action litigation, see generally Wolff (2005).

will not have sufficient assets available to satisfy the damage claims of all opt-out plaintiffs.

Third, we could relax the symmetry assumptions. For instance, we could consider the possibility that the class may enjoy enhanced bargaining as compared to individual plaintiffs (Silver 2000; Che 2002). We also could imagine that a plaintiff's bargaining power may be a function of the probability that it would prevail at trial.

Fourth, we could extend the model to give class members a second opportunity to opt out in stage 2 in the event of a proposed settlement of the class action. Extending the model to include a second opt-out would be consistent with the 2003 amendments to Rule 23, which, among other things, authorizes the court to refuse to approve a settlement in a Rule 23(b)(3) class action unless it affords class members a second opportunity to opt out after the terms of the settlement are known (see Fed. R. Civ. P. 23(e)(3); Advisory Committee's Notes to Rule 23).

Lastly, we could generalize the model to cover all possible allocations rules (i.e., all degrees of damage averaging). Technically, this would entail modeling the recovery to a plaintiff as a linear combination of his own damage claim and the average damage claim of the class. We also could modify or generalize the model to apply to or encompass other nonmandatory claim aggregation mechanisms.

APPENDIX

DISTRIBUTION OF $\Delta(W)$

Let $\varepsilon, \mu \sim Uniform(a, b)$ and define $\Delta = \varepsilon - \mu$. It can be shown that the probability density function of Δ is

$$f_\Delta(z) = \begin{cases} \frac{z+(b-a)}{(b-a)^2} & a - b \leq z \leq 0 \\ \frac{(b-a)-z}{(b-a)^2} & 0 < z \leq b - a \\ 0 & \text{otherwise} \end{cases} .$$

It follows that the cumulative distribution function of Δ is

$$F_\Delta(z) = \begin{cases} 0 & z < a - b \\ \frac{1}{2}\left(\frac{z+(b-a)}{(b-a)}\right)^2 & a - b \leq z \leq 0 \\ 1 - \frac{1}{2}\left(\frac{(b-a)-z}{(b-a)}\right)^2 & 0 < z \leq b - a \\ 1 & z \geq b - a \end{cases} .$$

If $W \in [0, \frac{1}{2}]$, then $a = -W$ and $b = W$, which implies

$$f_{\Delta(W)}(z) = \begin{cases} \frac{z+2W}{4W^2} & -2W \leq z \leq 0 \\ \frac{2W-z}{4W^2} & 0 < z \leq 2W \\ 0 & \text{otherwise} \end{cases}$$

and

$$F_{\Delta(W)}(z) = \begin{cases} 0 & z < -2W \\ \frac{1}{2}\left(\frac{z+2W}{2W}\right)^2 & -2W \leq z \leq 0 \\ 1 - \frac{1}{2}\left(\frac{2W-z}{2W}\right)^2 & 0 < z \leq 2W \\ 1 & z > 2W \end{cases} .$$

If $W \in [\frac{1}{2}, 1]$, then $a = W - 1$ and $b = 1 - W$, which implies

$$f_{\Delta(W)}(z) = \begin{cases} \frac{z+2(1-W)}{4(1-W)^2} & 2(W-1) \leq z \leq 0 \\ \frac{2(1-W)-z}{4(1-W)^2} & 0 < z \leq 2(1-W) \\ 0 & \text{otherwise} \end{cases}$$

and

$$F_{\Delta(W)}(z) = \begin{cases} 0 & z < 2(W-1) \\ \frac{1}{2}\left(\frac{z+2(1-W)}{2(1-W)}\right)^2 & 2(W-1) \leq z \leq 0 \\ 1 - \frac{1}{2}\left(\frac{2(1-W)-z}{2(1-W)}\right)^2 & 0 < z \leq 2(1-W) \\ 1 & z > 2(1-W) \end{cases}.$$

The density of $\Delta(W)$ is a symmetric tent (centered at $z = 0$) whose peak decreases to 1 as W increases from 0 to 0.5 and then increases as W increases from 0.5 to 1. Similarly, the distribution of $\Delta(W)$ is a symmetric "S" (through $F_{\Delta(W)}(0) = 0.5$) whose slope decreases as W increases from 0 to 0.5 and then increases as W increases from 0.5 to 1. Furthermore, it can be shown that $F_{\Delta(W)}(z)$ is continuous in W.

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CHAPTER 3

ANALOGICAL LEGAL REASONING: THEORY AND EVIDENCE

How do judges reason about the law? There are many theories. The canonical theory is that judges reason by analogy from case to case (Levi 1949; Weinreb 2005). This method of reasoning is known as analogical legal reasoning to jurisprudence scholars and case-based legal reasoning to scholars in the field of artificial intelligence and law.[1]

In its purest form, analogical legal reasoning (ALR) involves reasoning directly from prior cases to the case at hand—the judge evaluates the similarities and differences between prior cases and the case at hand and reaches a decision through application of the principle that like cases should be treated alike (Alexander and Sherwin 2008). Notably, ALR operates without invoking a legal rule that governs the decision in the case at hand (Sunstein 1993, 1996).[2]

A leading alternative theory is that judges reason deductively from legal rules (Alexander and Sherwin 2008; Schauer 2009).[3] In other words, they engage in rule-based legal reasoning (RLR). In its purest form, RLR operates without reference to prior cases—the judge simply applies the governing legal rule to the case at hand. At most, the judge uses prior cases to infer (perhaps abductively or inductively) the governing legal rule; however, she does not reason directly from case to case.

Both ALR and RLR constitute "legalist" theories of judicial behavior (Posner 2008). According to the legalist theory, "judges decide cases through systematic

[1] Some commentators argue that its use of analogy makes legal reasoning a distinctive form of reasoning (e.g., Fried 1981; Weinreb 2005). The mystical notion that legal reasoning is a distinctive form of reasoning was famously articulated by Sir Edward Coke, the Chief Justice of England, who denied the authority and competence of the King of England to render legal judgments on the grounds that legal questions "are not to be decided by natural reason but by the artificial reason and judgment of law" (*Prohibitions Del Roy*, 77 Eng. Rep. 1342 (1607)).

[2] On the different forms of analogical legal reasoning, see generally Macagno and Walton (2009).

[3] See also Westen (1982), Eisenberg (1988), Posner (1990, 1995, 2006, 2008), Schauer (1991), and Alexander (1996, 1998).

application of the external, objective sources of authority that classically comprise the law" (Cross 2003).[4] Although the legalist theory is the traditional theory of judicial behavior in legal circles, it has many critics. Perhaps the leading criticism of the legalist theory is that it suffers from theoretical indeterminacy (Cross 2003), presumably due to the paucity of formal models in the jurisprudence literature. ALR has been especially targeted by critics, with one commentator complaining that "it is infrequently described with any rigor or care" (Alexander 1996).[5]

This chapter has two objectives. The first objective is to offer a formal model of ALR. I model ALR as similarity-weighted averaging of prior outcomes. More specifically, the model posits that the outcome in the case at hand is a weighted average of the outcomes of prior cases, where the weights are a function of the fact similarity and precedential authority of prior cases. The full specification of the model appears in Section 3.1. The main theoretical result of the chapter is an axiomatization of a key feature of the model, which appears in Section 3.2.

The ALR model is closely related to the "empirical similarity" model of Gilboa et al. (2006), as well as the wider literature on case-based decision theory.[6] Case-based decision theory is an axiomatic model of reasoning by analogy to past cases (Gilboa and Schmeidler 2001).[7] Empirical similarity theory is a closely-related axiomatic model in which assessments are made according to similarity-weighted

[4] Of course, there are many other theories of judicial behavior. Posner (2008) identifies no fewer than nine theories, including most notably the legalist theory, the attitudinal theory, which posits that judges decide cases according to their ideological preferences (e.g., Segal and Spaeth 1993, 2002), and the economic and strategic theories, which posit that judges decide cases strategically, taking into account the responses of other actors, to promote their ideology (e.g., Epstein and Knight 1998; Smith and Tiller 2002), enhance their reputation or career prospects (e.g., Miceli and Coşgel 1994; Levy 2005), or further some other specified objective.

[5] Notable exceptions include Sunstein (1993, 1996), Brewer (1996), and Weinreb (2005).

[6] In case-based decision theory, the term "case" is used generically; it does not refer to a legal case.

[7] See also Gilboa and Schmeidler (1995, 1996, 1997, 2000, 2002, 2003) and Gilboa et al. (2002). Case-based decision theory was inspired by work on case-based reasoning in artificial intelligence (Riesbeck and Schank 1989) and harkens back to the notion that all human "reasonings concerning matter of fact are founded on a species of Analogy" (Hume 1748).

averages of prior assessments (Gilboa et al. 2006).[8] I compare and contrast the ALR model and empirical similarity theory in the course of specifying the ALR model in Section 3.1.

Empirical similarity theory is closely related to various methods in computer science, statistics, and related fields, including, most notably: kernel methods (Pagan and Ullah 1999), which are commonly used in nonparametric estimation; nearest neighbor methods (Dasarathy 1991; Devroye et al. 1996), which are commonly used in machine learning and pattern recognition; and conditional autoregressive (CAR) and simultaneous autoregressive (SAR) models (Banerjee et al. 2004), which are commonly used in the analysis of areal and other spatial data.[9] In addition, ALR is studied in the artificial intelligence and law literature, which contains a number of computational models of case-based adjudication (Rissland 1990; Rissland et al. 2003, 2006; Ashley and Brüninghaus 2006), as well as various theoretical models that are directed towards providing algorithmic or logical underpinnings for the computational models (Bench-Capon et al. 2004; Sartor 2005; Walton 2005; Bench-Capon and Prakken 2006; Bench-Capon et al. 2009).

The second objective of the chapter is to empirically evaluate the ALR model by testing whether it has more explanatory power than a simple RLR model. For a simple model of RLR, I turn to fractional polynomial regression (Royston and Altman 1994).[10] A fractional polynomial is an extension of a conventional polynomial that allows for noninteger and negative powers. Fractional polynomial regression is a flexible parametric method for approximating unknown functions using few parameters. Under the view that legal rules are functions (which map facts to

[8] See also Billot et al. (2005), Gayer et al. (2007), Billot et al. (2008), Lieberman (forthcoming), and Gilboa et al. (forthcoming).

[9] I expand upon the connection between empirical similarity theory and kernel regression in Section 3.3.1. For discussions of the relationship between empirical similarity theory, on the one hand, and nearest neighbor methods and conditional autoregressive models, on the other hand, see Lieberman (forthcoming) and Gilboa et al. (forthcoming).

[10] See also Royston and Altman (1997) and Royston and Sauerbrei (2008).

outcomes),[11] fractional polynomial regression provides a flexible yet parsimonious method for modeling legal rules.[12] The full specification of the RLR model appears in Section 3.3.

Using data on U.S. maritime salvage cases, I compare the ALR and RLR models according to their Bayesian information criteria (BIC).[13] Proposed by Schwarz (1978), BIC is a standard criterion for comparing and selecting among nonnested models. The maximum likelihood estimates for both models and the results of the BIC test appear in Section 3.3. The main conclusion is that the RLR model fits the data better than the ALR model.

As a supplement to the main empirical analysis, Section 3.3.4 presents a regression tree analysis of the maritime salvage cases. Regression tree analysis is a nonparametric method for analyzing the relationship between categorical or continuous independent variables and a continuous dependent variable (Bierman et al. 1984).[14] Although the regression tree analysis does not shed light directly on the question of which model better fits the data, it serves as a robustness check of the coefficient estimates for both models.

In Section 3.4, I discuss implications and limitations of the empirical analysis. I also discuss a conceptual issue that underlies the enterprise of the chapter, namely the extent to which ALR and RLR are theoretically distinct methods of legal reasoning. Concluding remarks appear in Section 3.5. The Appendix provides an overview of U.S. maritime salvage law.

[11] See, e.g., Kornhauser (1992a,b), Cameron et al. (2000), Cameron and Kornhauser (2005, 2009), and Kastellec (forthcoming).

[12] The use of mathematical and statistical methods to model legal rules is the enterprise of the fact-pattern analysis literature in political science (Kort 1957, 1963, 1968, 1973; Kort and Mars 1957; Mackaay and Robillard 1974; Segal 1984; Cameron and Kornhauser 2005; Kastellec forthcoming).

[13] In Section 3.3.3, I explain why U.S. maritime salvage cases provide a fertile testing ground for comparing the ALR and RLR models.

[14] A closely related method—classification tree analysis—is used when the dependent variable is categorical. Kastellec (forthcoming) conducts classification tree analysis of search and seizure cases decided by the U.S. Supreme Court and confession cases decided by the Courts of Appeals.

3.1 A MODEL OF ANALOGICAL LEGAL REASONING

Let $\mathcal{K}$ denote the set of judges or *courts* in the legal system. The courts in $\mathcal{K}$ are ordered in accordance with the hierarchy of courts in the legal system. Let $\mathcal{Q}$ denote the set of *questions* of law that may be presented to a court. For each question $q \in \mathcal{Q}$, there exists a set of *conclusions* of law $\mathcal{Y}_q$ that a court may reach with respect to question q and a vector of *issues* of fact $\varphi_q = (\varphi_{q1}, ..., \varphi_{qn})$ that the court must resolve in order to reach a conclusion with respect to question q.[15] For each issue φ_{qi}, there exists a set of *findings* of fact Φ_{qi} that the court may make with respect to issue φ_{qi}. Accordingly, each question $q \in \mathcal{Q}$ induces a *fact space* $\Phi_q = \Phi_{q1} \times \cdots \times \Phi_{qn}$. Each element $\phi = (\phi_1, ..., \phi_n) \in \Phi_q$ is a *fact pattern.* Given question q, the set of conclusions $\mathcal{Y}_q$, the vector of issues φ_q, and the fact space Φ_q are known and unique. A *case* involving question q is a triple $c = (\phi, \kappa, y)$, where $\phi \in \Phi_q$, $\kappa \in \mathcal{K}$, and $y \in \mathcal{Y}_q$. Define $x = (\phi, \kappa)$ as the *inputs* and y as the *outcome* of the case. The set of all possible cases involving question q is $\mathcal{C}_q = (\Phi_q \times \mathcal{K}) \times \mathcal{Y}_q$. I shall assume throughout the chapter that the inputs and outcomes of cases are or may be represented as real variables: $\Phi_q = \mathbb{R}^n_+$, $\mathcal{K} = \mathbb{R}_+$, and $\mathcal{Y}_q = \mathbb{R}$.

At time $t \in \mathbb{N}_{++}$, a court is presented with question q and a body of evidence. Based on the evidence, the court makes findings of fact $\phi_t \in \Phi_q$ with respect to issues φ_q. The court has access to a q-relevant *case history* $C_t = (c_1, ..., c_{t-1})$, where each $c_j = (x_j, y_j) \in \mathcal{C}_q$ is a prior case involving question q. How the court reaches its conclusion y_t depends on whether the court engages in ALR or RLR. What fundamentally distinguishes ALR and RLR is that under ALR the outcome of the case at hand is a function of the inputs of the case at hand as well as the history of prior cases, $y_t = Y(x_t, C_t)$, whereas under RLR the outcome of the case at hand is a function of the inputs only, $y_t = Y(x_t)$.[16]

[15] Note that n (the dimension of φ_q) is a function of q.

[16] Stated another way, under RLR the outcome depends on a bounded number of parameters,

I model ALR as similarity-weighted averaging of prior outcomes. Formally,

$$y_t = Y(x_t, C_t) = \sum_{j<t} \left(\frac{s(x_t, x_j)}{\sum_{j<t} s(x_t, x_j)} \right) y_j, \tag{3.1}$$

where

$$s(x_t, x_j) = \exp(-\mu(x_t, x_j)), \tag{3.2}$$

$$\mu(x_t, x_j) = v(x_t, x_j)\, d(\phi_t, \phi_j), \tag{3.3}$$

$$v(x_t, x_j) = \begin{cases} \cos\left(\arctan\left(\frac{\beta}{d(\phi_t,\phi_j)}\right)\right) & \text{if } \phi_t \neq \phi_j \ \&\ \kappa_t < \kappa_j \\ 1 & \text{otherwise} \end{cases}, \quad \beta \geq 0, \tag{3.4}$$

and

$$d(\phi_t, \phi_j) = \sqrt{\sum_{i=1}^{n} \omega_i (\phi_{ti} - \phi_{ji})^2}. \tag{3.5}$$

The model posits that the outcome y_t in the case at hand is a weighted average of the outcomes $y_1, ..., y_{t-1}$ of prior cases. The weight placed on the outcome y_j of a prior case depends on the degree to which the inputs x_j of the prior case are similar to the inputs x_t of the case at hand. The degree of input similarity is given by s. The greater is the input similarity of a prior case, the greater is the weight given to the outcome of the prior case in the determination of the outcome of the case at hand. Hence, I interpret s as measuring the *precedential influence* of a prior case on the case at hand. I assume that input similarity—and, therefore, precedential influence—is an exponentially decaying function of the distance μ from the inputs of the prior case to the inputs of the case at hand.[17]

whereas under ALR the number of parameters increases with the size of the prior case history (cf. Gayer et al. 2007).

[17]The assumption that influence decays exponentially with distance seems rather natural, and appears in other contexts as well (Bolhuis et al. 1986; Nosofsky 1986; Shephard 1987; Glaeser et al. 2003; Billot et al. 2008). Several studies provide evidence of exponential decay with time of the precedential influence of legal cases in U.S. federal courts (Post and Eisen 2000; Fowler and Jeon 2008; Black and Spriggs II 2009).

In turn, I assume that input distance is a proportional function (with nonconstant proportionality factor v) of the weighted Euclidean distance d between the facts ϕ_j of the prior case and the facts ϕ_t of the case at hand.[18] The proportionality factor v is less than one if the prior case was decided by a superior court ($\kappa_t < \kappa_j$) and equals one otherwise. All else equal, therefore, prior cases decided by a superior court receive greater weight—and, therefore, have more influence—than prior cases decided by a parallel or inferior court in the determination of the outcome of the case at hand. The size of this influence advantage, however, is smaller the greater is d (the distance between the prior case and the case at hand in fact space); how much smaller is determined by the shape parameter β.

The notion of similarity-weighted averaging as a model of analogical reasoning was introduced by Gilboa et al. (2006), who provided an axiomatization of similarity-weighted averaging with a generic similarity function. Although they did not specify a particular similarity function or even a particular functional form, Gilboa et al. (2006) were interested in similarity functions that depend on a weighted Euclidean distance. The notion of a similarity function that decays exponentially as a function of distance was introduced by Billot et al. (2008), who provided an axiomatization of an exponential similarity function based on a generic metric (i.e., a symmetric distance function). Billot et al. (2008) also axiomatized an exponential similarity function based on a weighted Euclidean distance, which is a special case of a metric.

In my model of ALR, the similarity function s is based on the input distance μ, which is a quasimetric (i.e., an asymmetric distance function) (see Section 3.2).[19] This generalization of the standard similarity function is instrumental to my pur-

[18] Note that the weights $\omega_1, ..., \omega_n$ in the weighted Euclidean distance d reflect the relative importance of the n issues of fact that the court must resolve in order to reach a conclusion with respect to the legal question at issue.

[19] Strictly speaking, μ is a quasimetric provided that the parameter β is sufficiently small (see Section 3.2).

poses,[20] because the precedential influence of a prior case in a legal system with hierarchical courts depends not only on the *fact similarity* of the prior case, which depends on the distance from the prior case in fact space, but also on the *precedential authority* of the prior case, which depends on the position in the judicial hierarchy of the court that decided the prior case. Although fact similarity is symmetric,[21] precedential authority is not symmetric. All else equal, the precedential authority of a case decided by a superior court is greater than the precedential authority of a case decided by a parallel or inferior court. Therefore, if the prior case was decided by a superior court, its influence on the case at hand ought to be greater than the influence of the case at hand on the prior case (under the counterfactual that the case at hand was decided before the prior case). Specifying a similarity function that is based on quasimetric allows the model to capture this distinctive feature of precedential influence in law.

Figure 3.1 displays the relationship in the model between precedential influence (s), fact similarity (d), and precedential authority (v).[22] As the figure illustrates, the precedential influence of a prior case is at its maximum ($s = 1$) when the facts of the prior case are identical to the facts of the case at hand ($\phi_t \neq \phi_j \Leftrightarrow d = 0$). As fact similarity decreases (i.e., as d increases), precedential influence decays exponentially at rate v—i.e., $s = \exp(-vd)$. The rate of decay differs depending on the precedential authority of the prior case. If the prior case was decided by a parallel or inferior court ($\kappa_t \geq \kappa_j$), the rate of decay equals one ($v = 1$). If, however, the prior case was decided by a superior court, the rate of decay is

[20] Like Gilboa et al. (2006) and Billot et al. (2008), Gayer et al. (2007), Lieberman (forthcoming), and Gilboa et al. (forthcoming) contemplate similarity functions based on a metric, usually a weighted Euclidean distance.

[21] That is, the distance in fact space from the prior case to the case at hand is equal to the distance from the case at hand to the prior case.

[22] Note that fact similarity and precedential authority are negatively related to d and v, respectively.

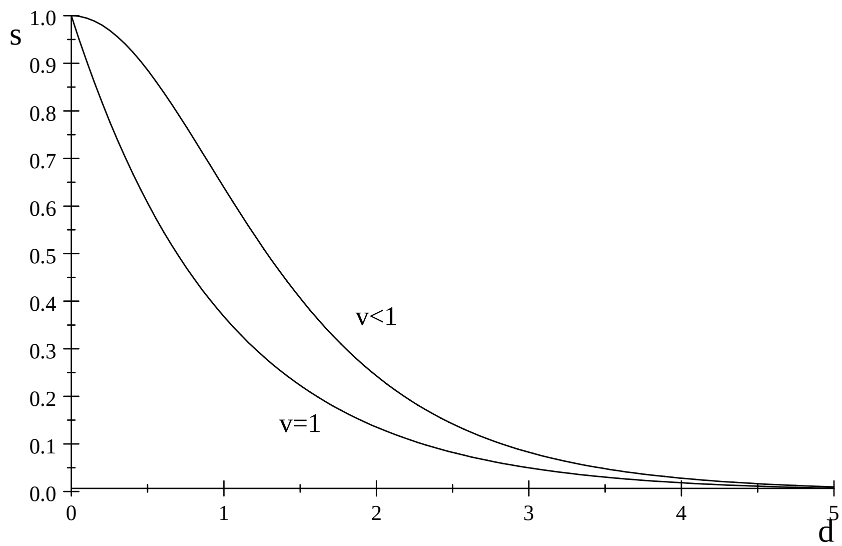

Figure 3.1: $s = \exp(-vd)$

$v = \cos(\arctan(\beta/d)) < 1$.[23] All else equal, therefore, the precedential influence of a prior case that was decided by a superior court is greater than precedential influence of a prior case that was decided by a parallel or inferior court. Moreover, the size of the influence advantage due to enhanced precedential authority (which, as noted above, is negatively related to v) increases with the degree of fact similarity (i.e., it increases as d decreases) at a rate determined by (and positively related to) the shape parameter β (see Figure 3.2).

[23] The specification of v is motivated as follows. For any two points $x_1, x_2 \in \mathbb{R}^n_+$, $x_1 \neq x_2$, a standard generic measure of the asymmetric distance from x_1 to x_2 is $f(\theta_{12})d(x_1,x_2)$, where: $d(x_1,x_2)$ is the Euclidean distance between x_1 and x_2; θ_{12} is the polar direction from x_1 to x_2; and $f(\theta)$ is a monotonically increasing function on $(0, \pi/2)$, typically chosen or normalized such that $f(0) = 0$ and $f(\pi/2) = 1$ (Drezner and Wesolowsky 1989). Turning to our setting, take any $x_j = (\phi_j, \kappa_j)$ and $x_t = (\phi_t, \kappa_t)$ in the input space $\Phi_q \times \mathcal{K} = \mathbb{R}^{n+1}_+$, where $\phi_t \neq \phi_j$ and $\kappa_t < \kappa_j$ (implying $x_j \neq x_t$). If we normalize the distance in $\mathcal{K} = \mathbb{R}_+$ (judicial hierarchy space) between superior and inferior courts to one (so $\kappa_j - \kappa_t = 1$), then $\theta_{jt} = \arctan(\beta/d(\phi_t, \phi_j))$ is the β-scaled polar direction from x_j to x_t, and $v(\theta_{jt}) = \cos(\theta_{jt})$ is just the standard quasimetric described above, where $f(\theta) = \cos(\theta)$ and d is the weighted Euclidean distance in $\Phi_q = \mathbb{R}^n_+$ (fact space).

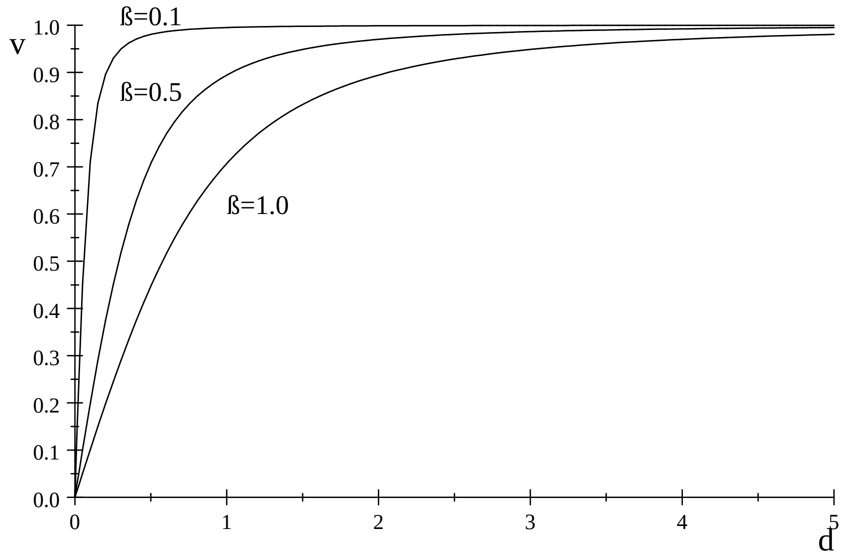

Figure 3.2: $v = \cos\left(\arctan\left(\frac{\beta}{d}\right)\right)$

3.2 THEORETICAL RESULTS

In this section I prove two theoretical results pertaining to the model.

3.2.1 Similarity as a Quasimetric

The first result is that the function $\mu : (\Phi_q \times \mathcal{K}) \times (\Phi_q \times \mathcal{K}) \rightarrow \mathbb{R}_+$, on which the similarity function $s : (\Phi_q \times \mathcal{K}) \times (\Phi_q \times \mathcal{K}) \rightarrow \mathbb{R}_{++}$ is based, is a *quasimetric* provided that the shape parameter β is sufficiently small. Recall the definition of a quasimetric:

Definition 3.1 (Quasimetric) *A function $\xi : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a* quasimetric *on $\mathbb{R}^n$ if for all $x, y \in \mathbb{R}^n$:*

(i) $\xi(x, y) = 0$ if $x = y$ (identity of indiscernibles);

(ii) $\xi(x, y) > 0$ if $x \neq y$ (positivity);

(iii) $\xi(x, z) \leq \xi(x, y) + \xi(y, z)$ for any $z \in \mathbb{R}^n$ (triangle inequality).

Note that a metric is a quasimetric that also satisfies symmetry: $\xi(x,y) = \xi(y,x)$. A quasimetric is not necessarily symmetric, i.e., in general $\xi(x,y) \neq \xi(y,x)$.

The following is a formal statement of the first result.

THEOREM 3.1 *For all $\phi, \psi \in \mathbb{R}^n$ and $\kappa, \iota \in \mathbb{R}$, with $w = (\phi, \kappa)$ and $z = (\psi, \iota)$, let*

$$\mu(w,z) = v(w,z)\, d(\phi,\psi),$$

where

$$v(w,z) = \begin{cases} \cos\left(\arctan\left(\frac{\beta}{d(\phi,\psi)}\right)\right) & \textit{if } \phi \neq \psi \textit{ \& } \kappa < \iota \\ 1 & \textit{otherwise} \end{cases}, \quad \beta \geq 0,$$

and d is a metric on $\mathbb{R}^n$. There exists $\rho > 0$ such that for all $\beta < \rho$, μ is a quasimetric on $\mathbb{R}^{n+1}$.

PROOF. First, take any $\beta \geq 0$. If $w = z$, then $\phi = \psi$ and $\kappa = \iota$. This implies $d(\phi,\psi) = 0$ and $v(w,z) = 1$, which in turn implies $\mu(w,z) = 0$. Thus, μ satisfies the identity of indiscernibles for every $\beta \geq 0$.

Next, suppose $\beta = 0$. Then $v(w,z) = 1$ (because $\arctan(0) = 0$ and $\cos(0) = 1$). This implies $\mu(w,z) = d(\phi,\psi)$. Because d is a metric, it follows that μ satisfies positivity and the triangle inequality for $\beta = 0$.

Finally, suppose $\beta > 0$. If $\phi = \psi$ or $\kappa \geq \iota$, then $v(w,z) = 1$ (by definition). This implies $\mu(w,z) = d(\phi,\psi)$. It follows that μ satisfies positivity and the triangle inequality for $\beta > 0$ when $\phi = \psi$ or $\kappa \geq \iota$.

If $\phi \neq \psi$ and $\kappa < \iota$, then $d(\phi,\psi) > 0$ (by positivity of d) and $v(w,z) \in (0,1)$ (because $\arctan(\theta) \in (0, \frac{\pi}{2})$ for $0 < \theta < \infty$ and $\cos(\theta) \in (0,1)$ for $0 < \theta < \frac{\pi}{2}$). It follows that $\mu(w,z) > 0$. (It also follows that $\mu(w,z) < d(\phi,\psi) = \mu(z,w)$.) Thus, μ satisfies positivity (but not symmetry) for $\beta > 0$ when $\phi \neq \psi$ and $\kappa < \iota$.

To complete the proof, we need to show that when $\phi \neq \psi$ and $\kappa < \iota$, there exists $\rho > 0$ such that μ satisfies the triangle inequality for every $\beta < \rho$. Take any $\vartheta \in \mathbb{R}^n$ and $\eta \in \mathbb{R}$, with $x = (\vartheta, \eta)$. Note that μ satisfies the triangle inequality iff $v\left(w, z\right) d\left(\phi, \psi\right) \leq v\left(w, x\right) d\left(\phi, \vartheta\right) + v\left(x, z\right) d\left(\vartheta, \psi\right)$.

Define $F(\beta) = v\left(w, x\right) d\left(\phi, \vartheta\right) + v\left(x, z\right) d\left(\vartheta, \psi\right) - v\left(w, z\right) d\left(\phi, \psi\right)$. Note that $F(0) = d\left(\phi, \vartheta\right) + d\left(\vartheta, \psi\right) - d\left(\phi, \psi\right)$ (because $v = 1$ for $\beta = 0$). Note further that $F(0) > 0$ (because d is a metric and $\phi \neq \psi$) and that F is continuous in β (because v is continuous in β). Because F is continuous in β, for every $\epsilon > 0$ there exists $\delta_\epsilon > 0$ such that $\beta < \delta_\epsilon$ implies $|F(\beta) - F(0)| < \epsilon$. Let $\epsilon = F(0) > 0$. It follows that there exists $\rho = \delta_{F(0)} > 0$ such that $\beta < \rho$ implies $F(\beta) > F(0) - \epsilon = 0$. It follows, in turn, that there exists $\rho > 0$ such that for all $\beta < \rho$, $v\left(w, x\right) d\left(\phi, \vartheta\right) + v\left(x, z\right) d\left(\vartheta, \psi\right) > v\left(w, z\right) d\left(\phi, \psi\right)$. ■

3.2.2 Axiomatization of an Exponential Similarity Function Based on a Quasimetric

The second result is an axiomatization of an exponential similarity function based on a quasimetric. The axiomatization closely follows Billot et al. (2008), which, as noted above, provided an axiomatization of an exponential similarity function based on a standard metric.

Let $\mathbb{C} = \cup_{t \geq 1} (\mathbb{R}^{n+2})^{t-1}$. Suppose there are given functions $Y : \mathbb{R}^{n+1} \times \mathbb{C} \rightarrow \mathbb{R}$ and $s : \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \rightarrow \mathbb{R}_{++}$ such that for all $x_t = (\phi_t, \kappa_t) \in \mathbb{R}^{n+1}$ and $C_t = (x_j, y_j)_{j<t} \in \mathbb{C}$,

$$y_t = Y\left(x_t, C_t\right) = \sum_{j<t} \left(\frac{s\left(x_t, x_j\right)}{\sum_{j<t} s\left(x_t, x_j\right)} \right) y_j \tag{3.6}$$

and s is normalized such that $s(x, x) = 1$ for all $x \in \mathbb{R}^{n+1}$.

I impose three axioms on Y.

Axiom 3.1 (Ray Monotonicity) *For all* $w, x, z \in \mathbb{R}^{n+1}$, $x \neq 0$, $Y(w, ((w + \lambda x, 1), (w + z, -1)))$ *is strictly decreasing in* $\lambda \geq 0$.

Axiom 3.1 considers a world with two prior cases, $c_1 = (\phi_1, y_1) = (w + \lambda x, 1)$ and $c_2 = (\phi_2, y_2) = (w + z, -1)$. In such a world, equation (3.6) would generate an outcome between y_1 and y_2, i.e., $Y \in [-1, 1]$. Axiom 3.1 states that as λ increases—whereby $\phi_1 = w + \lambda x$ moves further away from w (along a ray through w), thereby becoming less similar to w—Y decreases, i.e., moves away from $y_1 = 1$ and toward $y_2 = -1$.

Axiom 3.2 (Ray Shift Invariance) *For case histories of the form* $C = (w + \alpha_j x, y_j)_{j<t} \in \mathbb{C}$, *where* $w, x \in \mathbb{R}^{n+1}$, $y_j \in \mathbb{R}$ $(j < t)$, *and* $\alpha_j \geq 0$ $(j < t)$, $Y\left(w, (w + (\alpha_j + \delta) x, y_j)_{j<t}\right) = Y\left(w, (w + \alpha_j x, y_j)_{j<t}\right)$ *for all* $\delta > 0$.

Axiom 3.2 considers a world in which the inputs of all prior cases lie on a ray through the inputs of the case at hand, w. Axiom 3.2 requires that a shift δ along this ray leaves the outcome Y unchanged.

Axiom 3.3 (Self-relevance) *For all* $w, x, z \in \mathbb{R}^{n+1}$, $Y(z, ((w, 1), (x, 0))) \leq Y(w, ((w, 1), (x, 0)))$.

Axiom 3.3 considers a world with two prior cases, $(w, 1)$ and $(x, 0)$. In such a world, equation (3.6) would generate an outcome $Y \in [0, 1]$. Axiom 3.3 requires that the outcome Y in a new case z be less than the outcome Y in a new case w. The idea is that any new case z must be judged less similar to w than w is to itself.

Theorem 3.2 states the second result.

Theorem 3.2 *The following are equivalent:*

(i) Y satisfies Axioms 3.1-3.3;

(ii) There exists a quasimetric μ on $\mathbb{R}^{n+1}$ such that $s(w,z) = \exp(-\mu(w,z))$ for all $w, z \in \mathbb{R}^{n+1}$.

Proof. First, take any $w, x, z \in \mathbb{R}^{n+1}$ with $x \neq 0$. Ray Monotonicity (Axiom 3.1) holds iff

$$\frac{s(w, w+\lambda x) - s(w, w+z)}{s(w, w+\lambda x) + s(w, w+z)}$$

is strictly decreasing in $\lambda \geq 0$. Let $f(\lambda) = s(w, w+\lambda x)$. It follows that Ray Monotonicity holds iff

$$\frac{2s(w, w+z) \cdot f'(\lambda)}{[f(\lambda) + s(w, w+z)]^2} < 0,$$

which holds iff $f'(\lambda) < 0$.

Next, take $C = (w + \alpha_j x, y_j)_{j<t} \in \mathbb{C}$, where $w, x \in \mathbb{R}^{n+1}$, $y_j \in \mathbb{R}$ $(j < t)$ and $\alpha_j \geq 0$ $(j < t)$. Ray Shift Invariance (Axiom 3.2) holds iff for all $\beta > 0$,

$$\sum_{j<t} \left(\frac{s(w, w+(\alpha_j+\beta)x)}{\sum_{j<t} s(w, w+(\alpha_j+\beta)x)} - \frac{s(w, w+\alpha_j x)}{\sum_{j<t} s(w, w+\alpha_j x)} \right) y_j = 0.$$

This holds iff for all α_j and β,

$$\frac{s(w, w+(\alpha_j+\beta)x)}{s(w, w+\alpha_j x)} = \frac{\sum_{j<t} s(w, w+(\alpha_j+\beta)x)}{\sum_{j<t} s(w, w+\alpha_j x)} = g_\beta,$$

where $0 < g_\beta \leq 1$ (by Ray Monotonicity). Let $f(\lambda) = s(w, w+\lambda x)$, $\lambda \geq 0$. It follows that Ray Shift Invariance is equivalent to $f(\alpha_j + \beta) = f(\alpha_j) g_\beta$ for all α_j and β. Observe that $f(0) = 1$; therefore, $f(\beta) = g_\beta$ for all β. Accordingly, Ray Shift Invariance holds iff $f(\alpha_j + \beta) = f(\alpha_j) f(\beta)$ for all α_j and β. Because f is continuous and positive (by continuity and positivity of s), it follows that Ray

Shift Invariance is equivalent to $f(\lambda) = \exp(\lambda h_x)$ for $\lambda \geq 0$, where $h_x \leq 0$ (by Ray Monotonicity) with equality iff $x = 0$ (by $s(w, w) = 1$). Defining $z = w + \lambda x$ and $\mu(w, z) = -\lambda h_x$, we conclude that Axioms 3.1-3.2 are equivalent to $s(w, z) = \exp(-\mu(w, z))$ for all $w, z \in \mathbb{R}^{n+1}$, where the function μ satisfies all the properties of a quasimetric, apart from the triangle inequality.

The last step is to show that Self-relevance (Axiom 3.3) holds iff μ satisfies the triangle inequality, $\mu(z, x) \leq \mu(z, w) + \mu(w, x)$ for all $w, x, z \in \mathbb{R}^{n+1}$. Take any $w, x, z \in \mathbb{R}^{n+1}$. Self-relevance holds iff $Y(z, ((w, 1), (x, 0))) \leq Y(w, ((w, 1), (x, 0)))$, which holds iff

$$\frac{s(z,w)}{s(z,w)+s(z,x)} \leq \frac{1}{1+s(w,x)},$$

which, in turn, holds iff $s(z, x) \geq s(z, w)\, s(w, x)$. Because $s(w, z) = \exp(-\mu(w, z))$ for all $w, z \in \mathbb{R}^{n+1}$, this holds iff

$$\exp(-\mu(z, x)) \geq \exp(-\mu(z, w) - \mu(w, x)),$$

which, in turn, holds iff $\mu(z, x) \leq \mu(z, w) + \mu(w, x)$. ∎

3.3 EMPIRICAL ANALYSIS

Given the result of Theorem 3.2, Axioms 3.1 through 3.3 may be interpreted as observable implications of similarity-weighted averaging with an exponential similarity function based on a quasimetric. However, the special case histories contemplated by the axioms are not ones that we would expect to observe in the real world. Therefore, evaluating the ALR model by testing the validity of the axioms is not a promising strategy.

I adopt an empirical approach to evaluating the ALR model. Namely, I test whether the ALR model has more explanatory power than a simple RLR model.

First, I embed the ALR model specified in Section 3.1 in a statistical model. Second, I turn to fractional polynomial regression for a simple model of RLR. Next, I describe the data on U.S. maritime salvage cases, explain why the data provide a fertile testing ground for comparing the ALR and RLR models, and describe the criterion according to which I compare the models (namely, the BIC). I then present the maximum likelihood estimates for both models and the results of the BIC test. Lastly, I present a regression tree analysis of the maritime salvage cases as a supplement to the main empirical analysis.

3.3.1 Empirical Specification of the ALR Model

The first step of the empirical analysis is to embed the ALR model specified in Section 3.1 in a statistical model. Following Gilboa et al. (2006) and its progeny,[24] I assume that y_1 is an arbitrary random variable and that for $t = 2, ..., T$,

$$y_t = Y\left(x_t, C_t; \theta_{ALR}\right) = \sum_{j<t} \left(\frac{s\left(x_t, x_j\right)}{\sum_{j<t} s\left(x_t, x_j\right)} \right) y_j + \varepsilon_t, \tag{3.7}$$

where $\varepsilon_t \overset{iid}{\sim} N\left(0, \sigma^2\right)$, s is defined by (3.2)-(3.5), and $\theta_{ALR} = \left(\beta, \omega_1, ..., \omega_n, \sigma^2\right)$. I estimate the $n+2$ parameter vector θ_{ALR} via maximum likelihood. The log-likelihood function is

$$l\left(\theta_{ALR}\right) = -\frac{t}{2}\log\left(2\pi\right) - \frac{t}{2}\log\left(\sigma^2\right) - \frac{y'S'Sy}{2\sigma^2},$$

[24]See also Gayer et al. (2007), Lieberman (forthcoming), and Gilboa et al. (forthcoming).

where $y = (y_1, ..., y_T)'$ and

$$\underset{(t\times t)}{S} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & 0 & 0 & \cdots & 0 \\ -\frac{s(x_3,x_1)}{\sum_{j<3} s(x_3,x_j)} & -\frac{s(x_3,x_2)}{\sum_{j<3} s(x_3,x_j)} & 1 & 0 & 0 & \cdots & 0 \\ -\frac{s(x_4,x_1)}{\sum_{j<4} s(x_4,x_j)} & -\frac{s(x_4,x_2)}{\sum_{j<4} s(x_4,x_j)} & -\frac{s(x_4,x_3)}{\sum_{j<4} s(x_4,x_j)} & 1 & 0 & \cdots & 0 \\ \vdots & & & & \ddots & & \\ -\frac{s(x_t,x_1)}{\sum_{j<t} s(x_t,x_j)} & \cdots & & & \cdots & -\frac{s(x_t,x_{t-1})}{\sum_{j<t} s(x_t,x_j)} & 1 \end{pmatrix}.$$

For the derivation of the log-likelihood function, as well as an explication of the asymptotic theory of model (3.7), which establishes a theoretical basis for simple hypothesis tests involving the model parameters, see Lieberman (forthcoming).

Before turning to the RLR model, let me say a few words about the relationship between model (3.7) and kernel regression.[25] Kernel regression assumes a data generating process of the form

$$y_i = g(x_i) + e_i, \; i = 1, ..., N, \tag{3.8}$$

where $e_i \overset{iid}{\sim} (0, \sigma^2)$ and g is an unknown function. A standard estimator for g is the Nadaraya-Watson estimator

$$\hat{g}(x) = \sum_{i\leq n} \left(\frac{K\left(\frac{x_i - x}{h}\right)}{\sum_{i\leq n} K\left(\frac{x_i - x}{h}\right)} \right) y_i, \tag{3.9}$$

where K is a kernel function (i.e., a non-negative function satisfying, among other regularity conditions, $\int K(z)dz = 1$) and h is a bandwidth parameter. Note that the right-hand side of equation (3.9) has the same form as the first term on the right-hand side of equation (3.7). Indeed, a direct mapping exists between them

[25] On kernel regression, see Pagan and Ullah (1999).

(Gilboa et al. forthcoming). Moreover, they operate in the same way. Each generates a new/predicted value of y by taking a weighted average of the observed values of y where the weights are a function the distance between the new/hypothesized x and the observed values of x. Notwithstanding these connections, however, there is an important distinction between model (3.7) and kernel regression. Kernel regression is a statistical technique that uses weighted averaging to estimate model (3.8), which assumes that the data are generated by a function (i.e., a rule), whereas model (3.7) assumes that that the data are generated by weighted averaging.[26]

3.3.2 Modeling RLR Using Fractional Polynomial Regression

The second step is to specify a simple RLR model. The essence of RLR is that it involves the application of a governing legal rule to the case at hand. The source of the governing legal rule is not important; for example, the rule may be stated in or inferred from a statute or regulation or it may be stated in or inferred from prior cases. What is important is that the court invokes the legal rule in determining the outcome in the case at hand.

A legal rule may be viewed as a function which map facts to outcomes.[27] This view suggests a simple model of RLR—the outcome in case at hand is a function of the facts of the case at hand, $y_t = Y(\phi_t)$. A pragmatic, parametric approach to estimating this unknown function is fractional polynomial regression (Royston and Altman 1994).[28] A fractional polynomial is an extension of a conventional polynomial that allows for noninteger and negative powers. In reliance on Taylor's theorem, conventional polynomials are often used to approximate unknown functions. However, polynomial regression generally involves a tradeoff between

[26] For more on the relationship between model (3.7) and kernel regression, see Gilboa et al. (2006, forthcoming) and Lieberman (forthcoming).

[27] See, e.g., Kornhauser (1992a,b), Cameron et al. (2000), Cameron and Kornhauser (2005, 2009), and Kastellec (forthcoming).

[28] See also Royston and Altman (1997) and Royston and Sauerbrei (2008).

flexibility (i.e., fit) and parsimony. Royston and Altman (1994) introduced fractional polynomial regression as a flexible parametric method for approximating unknown functions using few parameters.

The standard multivariable fractional polynomial (MFP) regression model may be expressed as

$$y_t = Y(\phi_t; \theta_{RLR}) = b_0 + \sum_{i=1}^{h} b_i \phi_{it} + \sum_{i=h+1}^{n} \sum_{j=1}^{m} b_{ij} \phi_{it}^{(p_j)} + \varepsilon_t, \; t = 1, ..., T, \qquad (3.10)$$

where $\varepsilon_t \overset{iid}{\sim} N(0, \sigma^2)$ and $\theta_{RLR} = (b_0, b_1, ..., b_h, b_{h+1,1}, ..., b_{h+1,m}, ..., b_{n1}, ..., b_{nm}, \sigma^2)$. The first h covariates, $\phi_1, ..., \phi_h$, are binary, categorical, or ordinal, and the remaining covariates, $\phi_{h+1}, ..., \phi_n$, are continuous. The powers $p_1, ..., p_m$ are chosen from a predefined set $\mathcal{P}$ according to the MFP algorithm developed by Sauerbrei and Royston (1999).[29] The round bracket notation signifies the Box-Tidwell transformation,

$$\phi_{it}^{(p_j)} = \begin{cases} \phi_{it}^{(p_j)} & \text{for } p_j \neq 0 \\ \ln \phi_{it} & \text{for } p_j = 0 \end{cases} .$$

The degree m is predefined by the researcher. The researcher also predefines two significance levels: α_1, which determines the critical value for variable selection; and α_2, which determines the critical value for model selection. The parameter vector θ_{RLR}, which has a maximum of $(n-h+1)m+h+1$ parameters, is estimated via maximum likelihood.

3.3.3 Data, Empirical Strategy, and Results

My empirical strategy is to compare the ability of the ALR and RLR models (models (3.7) and (3.10), respectively) to explain the time series of U.S. maritime salvage cases. Under federal maritime law, a salvor of imperiled maritime prop-

[29] See also Royston and Sauerbrei (2008).

erty on navigable waters is entitled to a monetary award from the owner.[30] There are two forms of maritime salvage: "contract" salvage and "pure" salvage. Contract salvage is rendered pursuant to a prior agreement. Pure salvage is rendered voluntarily in the absence of a contract. The data include only pure salvage cases.

In the United States, the federal courts have exclusive admiralty jurisdiction in cases involving claims for salvage awards. There are three elements of a valid pure salvage claim: (i) a marine peril; (ii) service rendered voluntarily (and not required by a preexisting duty or contract); and (iii) success in whole or in part. The peril need not be immediately impending—reasonable apprehension of danger is sufficient. In addition, although a party may render salvage services without the request of the owner, it may not force its services upon an owner who refuses assistance. Finally, under the "no cure-no pay" rule, there can be no salvage claim if the property is lost, notwithstanding the efforts of the putative salvors.

In the case of a valid pure salvage claim, the court determines the proper award according to six factors enumerated by the Supreme Court in *The Blackwall*, 77 U.S. (10 Wall.) 1 (1869):

(1) the labor expended by the salvors in rendering the salvage service;

(2) the promptitude, skill, and energy displayed in rendering the service and saving the property;

(3) the value of the property employed by the salvors in rendering the service, and the danger to which such property was exposed;

(4) the risk incurred by the salvors in securing the property from the impending peril;

[30] The following is a bare bones description of U.S. maritime salvage law. A more detailed overview appears in the Appendix.

(5) the value of the property saved; and

(6) the degree of danger from which the property was rescued.

There is no precise formula for computing a salvage award on the basis of the *Blackwall* factors. The court has considerable discretion in weighing the factors and making its determination on a case-by-case basis. The award, however, is limited by the value of the property saved.

The data comprise 130 pure salvage cases from 1880 to 2007.[31] For each case, the data record the salvage award (in thousands of 1980 dollars),[32] the court's findings of fact with respect to each of the six *Blackwall* factors (high = 1 or low = 0),[33] and the position of the court in the judicial hierarchy (circuit court = 1 or district court = 0).[34] Tables 3.1 and 3.2 provide descriptive statistics. Table 3.1 displays summary statistics for each variable. For instance, it shows that the salvage awards range from \$320 to \$1,865,000 with a mean award of \$122,000; the values of the salvaged property range from \$1,2000 to \$23,400,000 with a mean value of \$2,340,000; the danger to the salvaged property was high in 59 percent of the cases; and 32 percent of the cases were finally adjudicated by a circuit court. Table 3.2 displays the mean salvage award expressed as a percentage of the value

[31] The cases were identified by two search methods. The first was "KeyCiting" and "Shepardizing" *The Blackwall* in Westlaw and LexisNexis, respectively. The second was searching three databases: Westlaw's Federal Maritime Law - Cases (FMRT-CS); LexisNexis' Admiralty Cases - Federal and State (MEGA); and American Maritime Cases (AMC), which is available on Westaw and LexisNexis. The searches were designed to locate all reported federal cases decided after December 31, 1869 and on or before December 31, 2007 that apply the *Blackwall* factors to determine a pure salvage award, whether or not the cases cited *The Blackwall.*

[32] Adjustments for inflation were made using Tom's Inflation Calculator, available at http://www.halfhill.com/inflation.html.

[33] I read each case and hand coded the data. For the vast majority of cases, it was straightforward to determine the salvage award and the court's findings of fact with respect to all six *Blackwall* factors. For a handful of cases, however, information necessary to determine one or more variables was missing; these cases are excluded from the data. Absent a good reason why a court's method of reasoning would be correlated with a case having missing information, there is no reason to believe that excluding these cases biases the results of the empirical analysis.

[34] For each case, the court is the court of final adjudication, and the data record the salvage award and findings of fact as determined by the court of final adjudication.

Table 3.1: Summary Statistics

	Variable	Mean	Std Dev	Min	Max
y	Salvage award	122	271	0.32	1,865
x_1	Labor expended by salvors	0.35	0.48	0	1
x_2	Skill displayed by salvors	0.58	0.49	0	1
x_3	Danger to salvors' property	0.26	0.44	0	1
x_4	Risk to salvors	0.25	0.43	0	1
x_5	Value of salvaged property	2,340	4,529	1.20	23,400
x_6	Danger to salvaged property	0.59	0.49	0	1
$court$	Circuit court indicator	0.32	0.57	0	1

Notes: 130 cases from 1880 to 2007. y and x^5 in thousands of 1980 dollars.

Table 3.2: Conditional Mean Award Percentages

Variable	Obs	Mean
$y/x5$	130	0.132
$y/x5$ if $x1 = 0$	85	0.089
$y/x5$ if $x2 = 0$	54	0.092
$y/x5$ if $x3 = 0$	96	0.104
$y/x5$ if $x4 = 0$	98	0.104
$y/x5$ if $x6 = 0$	53	0.087
$y/x5$ if $x1 = 1$	45	0.212
$y/x5$ if $x2 = 1$	76	0.160
$y/x5$ if $x3 = 1$	34	0.211
$y/x5$ if $x4 = 1$	32	0.216
$y/x5$ if $x6 = 1$	77	0.162

of the salvaged property, as well as conditional means given different findings of fact. For instance, it shows that the (unconditional) mean award percentage is 13.2 percent; the mean award percentage for cases in which the danger to the salvaged property was low is 8.9 percent; and the mean award percentage for cases in which the labor expended by the salvors was high is 21.2 percent.

There are several reasons why maritime salvage cases provide a fertile testing ground for comparing the ALR and RLR models. First, the outcome (the salvage award) is a continuous variable (a dollar amount) and the inputs (the *Blackwall* factors) are well defined and stable over time.[35] Second, awards in maritime salvage cases arguably are apolitical legal questions. Moreover, it is hard to imagine that a maritime salvage case is an opportunity for a judge to advance strategic goals such as career advancement. Thus, if there is any setting in which we should expect "legalist" models of judicial behavior to be operative (and other models such as attitudinal or strategic models to be inoperative), it is maritime salvage cases. Third, the law of maritime salvage is federal common law, and, as noted above, federal courts have exclusive jurisdiction in cases involving claims for salvage awards. Accordingly, state variation in law or courts is not an issue. Fourth, it seems reasonable to treat the federal courts as a single adjudicative body for purposes of maritime salvage cases: there is no split among the circuits (*Blackwall* is controlling precedent for all circuits); there are no specialty courts for maritime cases; and it generally is believed that federal courts are reasonably uniform in quality. Lastly, as noted above, although the criteria for determining a maritime salvage award are well defined and stable through time, there is no explicit formula or rule. This leaves open the possibility that courts are engaging in ALR or RLR.

I compare the ALR and RLR models according to their Bayesian information

[35] In the words of the U.S. Court of Appeals for the Ninth Circuit, the *Blackwall* factors "have weathered the storms of the past century" (*Saint Paul Marine Transp. Corp. v. Cerro Sales Corp.*, 505 F.2d 1115 (9th Cir. 1974)).

criteria, or BIC (Schwarz 1978). For a given model, the BIC is

$$BIC = l(\hat{\theta}) - \frac{1}{2} k \log T,$$

where $l(\hat{\theta})$ is the maximized value of the likelihood function for the model, k is the number of model parameters (i.e., the dimension of the parameter vector θ), and T is the number of observations. The basic idea behind the BIC is that it selects the model with the highest likelihood value (or best fit), subject to a penalty for lack of parsimony (or overfitting).

Tables 3.3 and 3.4 present the maximum likelihood estimates and BIC for the ALR model and the benchmark RLR model, respectively. (Note that in the benchmark RLR model, I set $\mathcal{P} = \{-2, -1, -0.5, 0, 0.5, 1, 2, ..., 5\}$, $m = 5$, and $(\alpha_1, \alpha_2) = (0.05, 0.05)$. Thus, although I allow for a fifth degree fractional polynomial (in the one continuos predictor, $\ln x_5$) with powers ranging from -2 to 5, the MFP algorithm selects a simple linear specification.[36]) In both models, the dependent variable is the natural logarithm of the salvage award and the independent variables are the court's findings of fact with respect to the six *Blackwall* factors (where the value of the salvaged property (factor 5) is log-transformed). The estimates for both models suggest that three factors are statistically significant to the determination of the salvage award—the labor expended by the salvors (factor 1), the value of the salvaged property (factor 5), and the danger to the salvaged property (factor 6)—with factors 5 and 6 commanding roughly equal weight and factor 1 commanding greater weight than factors 5 and 6.[37]

[36]As a check of this selection, I estimated three alternative specifications of the RLR model. The first was a fifth degree convential polynomial (in $\ln x_5$). In the second alternative specification, I set $\mathcal{P} = \{-2, -1, -0.5, 0, 0.5, 1, 2, ..., 8\}$, thereby enlarging the set of powers. In the third alternative specification, I set $\alpha_2 = 1$, thereby forcing the MFP algorithm mto fit the best possible fifth degree fractional polynomial (in $\ln x_5$). None of the alternative specifications outperformed the benchmark model.

[37]In addition, note that the estimate for the shape parameter β in the ALR model is sufficiently small such that μ is a quasimetric on the input space.

Table 3.3: ALR Model (dependent variable: ln y)

	Variable	Coeff	Std Err
x_1	Labor expended by salvors	16.49**	5.56
x_2	Skill displayed by salvors	1.60	2.54
x_3	Danger to salvors' property	6.16	3.95
x_4	Risk to salvors	0.71	2.25
$\ln x_5$	Value of salvaged property	4.97**	1.50
x_6	Danger to salvaged property	3.38*	1.75
β	Shape parameter for v	0.00	0.29
	Loglikelihood	-244.16	
	BIC	-263.63	

Notes: *Significant at 5% level. **Significant at 1% level. $\hat{\beta} = 1.3821 \times 10^{-7}$.

Table 3.4: Benchmark RLR Model (dependent variable: ln y)

	Variable	Coeff	Std Err
x_1	Labor expended by salvors	1.02**	0.20
x_2	Skill displayed by salvors	0.23	0.20
x_3	Danger to salvors' property	0.35	0.27
x_4	Risk to salvors	0.16	0.29
$\ln x_5$	Value of salvaged property	0.60**	0.05
x_6	Danger to salvaged property	0.59**	0.20
	Constant	9.41**	0.15
	Loglikelihood	-177.99	
	BIC	-197.46	

Note: **Significant at 1% level.

While interesting in their own right, the coefficient estimates are not my main concern. Rather, my main concern is the result of the BIC test. The BIC for the ALR model is -263.63, whereas the BIC for the benchmark RLR model is -197.46. This suggests that that the RLR model fits the data better than the ALR model. I discuss the implications of this result in Section 3.4.

3.3.4 Regression Tree Analysis

The final step of the empirical analysis is to perform a regression tree analysis of the maritime salvage cases. Regression tree analysis is a nonparametric method for analyzing the relationship between categorical or continuous independent variables and a continuous dependent variable (Bierman et al. 1984). I present the regression tree analysis as a supplement to the main empirical analysis. Although the regression tree analysis does not shed light directly on the question of whether the ALR or RLR model better fits the data, it serves as a robustness check of the coefficient estimates for both models.

Table 3.5 summarizes the regression tree model. As before, the dependent variable is the natural logarithm of the salvage award and the independent variables are the court's findings of fact with respect to the six *Blackwall* factors (where the value of the salvaged property (factor 5) is log-transformed). The tree growing method partitions the data to minimize within-node variance ("impurity"),[38] subject to four criteria/limitations: (i) only binary splits are allowed; (ii) the min-

[38] Formally, the measure of impurity is least squares deviation (LSD), and is computed as

$$\frac{1}{N(\tau)} \sum_{t \in \tau} \left[y_t - \bar{y}(\tau) \right]^2 ,$$

where is $N(\tau)$ is the number of cases in node τ, y_t is the salvage award in case t, and

$$\bar{y}(\tau) = \frac{1}{N(\tau)} \sum_{t \in \tau} y_t$$

is the mean salvage award for cases in node τ.

Table 3.5: Summary of Regression Tree Model

Dependent variable:	$\ln y$
Independent variables:	x_1, x_2, x_3, x_4, $\ln x_5$, and x_6
Growing method:	CART
Impurity measure:	LSD
Split type:	binary
Minimum improvement:	0.0001
Maximum tree depth:	5
Minimum cases per node:	10

imum decrease in impurity required to split a node is is 0.0001; (iii) the maximum number of levels beneath the root node is five; and (iv) the minimum number of cases in each node is ten.

Figure 3.3 presents the regression tree analysis.[39] Like the ALR and RLR model estimates, the regression tree analysis suggests that three factors are key to the determination of the salvage award: the labor expended by the salvors (factor 1), the value of the salvaged property (factor 5), and the danger to the salvaged property (factor 6). Unlike the ALR and RLR model estimates, however, the regression tree analysis suggests that the value of the salvaged property (factor 5) is the most important factor.[40] Moreover, the regression tree analysis suggests how the factors interact. When the value of the salvaged property is low, this fact alone appears to dictate a small award. A somewhat larger award is made when the value of the salvaged property is moderate and the labor expended by the salvors is low. Higher awards are granted when either the value of the salvaged property is moderate and the labor expended by the salvors is high or the value of the salvaged property is high and the danger to the salvaged property is low. The highest awards occur when the value of the salvaged property and the danger to

[39] The risk estimate is the mean within-node variance across the terminal nodes. Note that the tree did not require pruning to avoid overfitting.

[40] A model importance analysis suggests factor 5 is five times as important as factor 1 or factor 6.

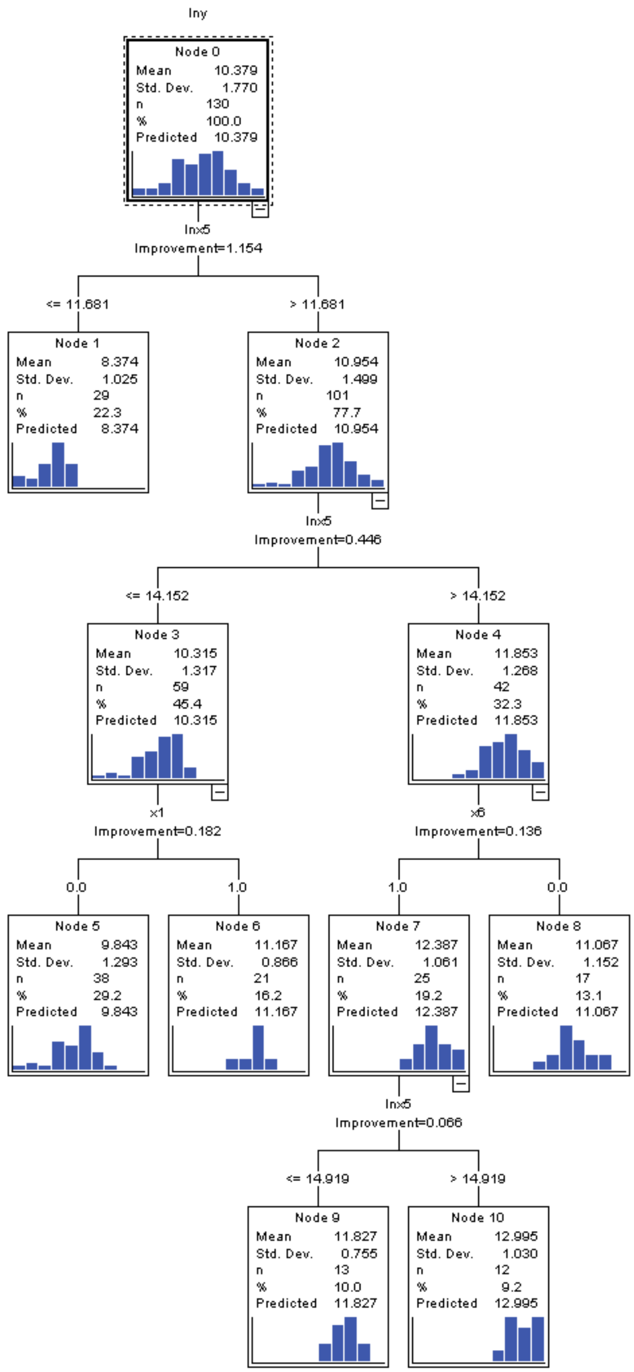

Figure 3.3: Regression Tree Analysis (risk estimate: 1.123)

the salvaged property are high.

3.4 IMPLICATIONS AND LIMITATIONS

The main conclusion of the empirical analysis is that the RLR model fits the data better than the ALR model. This conclusion is based on a comparison of the BIC of the two models. The key implication is that it is more likely that the data were generated by rule-based legal reasoning than by analogical legal reasoning. This implication, however, is subject to several limitations.

First, data on the inputs and outcomes of legal cases provides only indirect evidence regarding the method of legal reasoning. Nevertheless, it arguably is the best available evidence. In many cases, a court's written opinion offers no direct evidence regarding the method of legal reasoning. Even in cases in which the court's opinion offers some direct evidence, it rarely is definitive and, in any event, it arguably is of little probative value.[41]

Second, the two models are simplistic representations of ALR and RLR in their purest forms. Thus, not only are they highly stylized, they are rather extreme. It is quite possible that a combination or hybrid model, perhaps along the lines of a mixed SAR model (Anselin 1988), may fit the data better than either model. Examination of such a hybrid model would be intriguing direction for future research.

Third, both models take a representative agent approach. The ALR model assumes that all judges are equipped with the same similarity function, whereas the RLR model assumes that all judges apply the same legal rule. Allowing for heterogeneous judges surely would be more realistic, although tractability would

[41] "As a rule, we conceive of the judge's writing of an opinion as a procedure in which he justifies his decision. The writing coincides neither necessarily nor realistically with the process by which he reaches his decision, the process of discovery" (Murray 1982). There are (at least) two reasons to think that a court might use the language of RLR to justify its decision even if it engages in ALR in reaching its decision. First, "the language of 'rules' is much more efficient and parsimonious than that of 'cases'" (Gilboa and Schmeidler 2000). Second, "[r]ules are excellent justification mechanisms" (Hunter 2001).

require making strong assumptions about the structure of such heterogeneity.

Fourth, the ALR is less flexible than the RLR model. The ALR model is quite rigid in terms of structure. It specifies a particular method of assessment (similarity-weighted averaging) as well as a very specific functional form for the similarity function (equations (3.2)-(3.5)). The RLR model is more flexible. A fractional polynomial can approximate any sufficiently smooth function. It is conceivable that a more flexible specification (or even just a different specification) of the ALR could outperform the RLR model. Future research could explore the robustness of the BIC test result to alternative specifications of the ALR model.

Lastly, the empirical analysis in Section 3.3 offers not a statistical hypothesis test but rather a model selection exercise. The objective is to choose the best of the two competing models, without regard to whether either model is false. As such, the result of the BIC test cannot be interpreted either as a rejection of the hypothesis that the data were generated by ALR nor as a failure to reject the hypothesis that the data were generated by RLR. Rather, it should be interpreted as statistical evidence favoring the RLR model over the ALR model.

A further limitation of the empirical analysis is that it speaks only to the ability of the ALR and RLR models to explain the awards in maritime salvage cases. It says nothing about the ability of either model to explain the outcomes of cases in other areas of law. Moreover, the fact that both models are legalist models of judicial behavior suggests that they may not be well suited to other areas of law, including, in particular, politically charged areas (to which we might expect attitudinal or strategic models to be better suited).

Finally, the import of the empirical analysis is subject to an underlying conceptual question regarding the theoretical distinction between "analogical" and "rule-based" methods. One might question the distinction at two levels. At a general level, one might ask, if judges, as a rule, decide new cases via similarity-weighted averaging of prior cases, then is this not a "rule-based" method? The answer is no. The question speciously trades on an ambiguity in the meaning of the word "rule" in ordinary language. When jurisprudence scholars refer to "rule-based" methods, they mean methods that entail invoking generalizations (Schauer 1991; Alexander and Sherwin 2008; Schauer 2009). They do not mean any method that judges generally or even invariably use to decide new cases.

At a deeper level, one might ask, does not the process of making similarity judgments entail invoking generalizations (and thereby render "rule-based" all putative "analogical" methods)? This question is the subject of active debate among jurisprudence scholars. Skeptics argue that judges cannot make similarity judgments without invoking generalizations (Dworkin 1997; Alexander and Sherwin 2008).[42] Alexander and Sherwin (2008) state the skeptical view thusly:

> "Our point is that [a judge] cannot *reason* that [two cases] should be decided alike because they similar. To reason that they should be decided alike, she must determine that they are *importantly* similar, and to reason that they are importantly similar, she must refer to some general proposition . . . . [I]n order to draw analogies . . . [a judge] must refer to some general proposition that supports the analogy. . . . [T]he rules or principles that govern similarity, rather than the outcome of the precedent case, determine the result of the new case."

Dworkin (1997) makes the point succinctly: "An analogy is a way of stating a conclusion, not a way of reaching one, and theory must do the real work."

Nonskeptics insist that judges can and do make similarity judgments without invoking generalizations (Kamm 1997; Hunter 2001; Weinreb 2005). In the words

[42] See also Eisenberg (1988), Eisenberg (1988), Greenwalt (1992), Posner (1990, 1995, 2006, 2008), and Alexander (1996, 1998). Milder skeptics include Levi (1949), Sunstein (1993, 1996), and Brewer (1996).

of Hunter (2001), "analogy is a one-to-one similarity comparison that requires no generalization to operative effectively." Responding directly to Dworkin, Kamm (1997) writes:

> "I disagree with Dworkin when he says that analogy is only a way of stating conclusions. Analogy can be a way of reaching a conclusion. The relevance of an analogous case can be clear, even if one does not have a theory that links the analogous case and the original case, and even if one is initially uncertain about what one may permissibly do in the analogous case. While we may need a theory to *explain* why case A is really more like case B than case C, we may still, without deep theoretical justification, see that case A is more like B than C and use that conclusion to help us find a solution to case A. Indeed, sometimes one reaches a conclusion about a case by way of an analogous case and still does not provide an adequate theoretical justification of one's position in either case."

Weinreb (2005) offers an extensive defense of the nonskeptical view. He rejects the skeptical view on two grounds. First, citing research in cognitive science and psychology, Weinreb posits that "the capacity for analogical reasoning is hard-wired in us," including the "idea of *relevant* similarity." Moreover, he asserts that analogical reasoning "cannot be assimilated or reduced" to rule-based reasoning because the latter depends on the ability to discern relevant similarity. According to Weinreb: "Unless one is able to identify an object as a member of a class despite its differences from other members of the class, no deductive inference is possible."

Second, Weinreb contends that the skeptics' argument "proves too much." He says:

> "By the same reasoning that would require a rule that makes the similarity on which an analogy rests relevant, so also would there have to be a rule for each and every one of the innumerable other similarities and dissimilarities between the two things compared. Otherwise, how would one know that beside the similarity to which the rule referred, there was not some other feature of one or both that also was relevant to the outcome, which would be changed accordingly?"

Weinreb accuses the skeptics of having it "backwards" when they argue that "the rule precedes and is essential to the validity of any analogy on which the decision rests." Instead, he avers that "the 'rule of the case' . . . is a generalized statement of the decision, not the predicate on which the decision rests," and that "[r]ather than the analogy depending on the rule, the rule depends on the analogy."

I offer the ALR model as a formal representation of the nonskeptical account of analogical legal reasoning. In doing so, I do not stake out a position in the debate between the skeptics and nonskeptics. Rather, I simply allow the possibility that the nonskeptical account is correct, and develop a formal representation using the apparatus of case-based decision theory, which embraces the nonskeptical view.[43] Under case-based decision theory, "the notion of similarity is primitive" (Gilboa and Schmeidler 2001).[44] A case-based decision maker does not engage in *explicit* induction, whereby one formulates general rules. Rather, she engages in *implicit* induction (Gilboa and Schmeidler 2000, 2001), whereby "similar past cases are implicitly generalized to bear upon future cases" (Gilboa and Schmeidler

[43]It is noteworthy that Alexander and Sherwin (2008) concede to Weinreb that if "it is in fact psychologically possible . . . to intuit important similarity without referrring to a supporting generalization, this decision is geneuinely analogical." As a normative matter, however, they lament that this is "a very impoverished view of judicial decision making, which we are reluctant to attribute to judges adjudicating in good faith."

[44]See also Gilboa and Schmeidler (2000). Like Weinreb, Gilboa and Schmeidler (2001) suggest that "[o]ur ability to discuss counterfactuals . . . relies on our subjective similarity judgments."

2000).[45] Similarity-weighting averaging is one example of implicit induction. Under similarity-weighted averaging, although the decision maker "does not explicitly resort to general rules and theories," she "can be viewed as someone who believes in a general rule of the form $Y = f(X^1, ..., X^m)$ but does not know the functional form of f and therefore attempts to estimate it by nonparametric techniques" (Gilboa et al. 2006).[46]

3.5 CONCLUDING REMARKS

The use of analogical reasoning in law is a central topic in the jurisprudence and artificial intelligence and law literatures. Contributing to these literatures, this chapter presents and empirically evaluates a formal model of analogical legal reasoning. The model posits that the outcome of the case at hand is a weighted average of the outcomes of prior cases, where the weights are a function of the fact similarity and precedential authority of prior cases. To evaluate the model, I test its ability to explain the outcomes in U.S. maritime salvage cases vis-à-vis a simple model of rule-based legal reasoning (for which I turn to fractional polynomial regression). Comparing their Bayesian information criteria, I find that the RLR model fits the data better than the ALR model. As a supplement to the main empirical analysis, I present a regression tree analysis of the maritime salvage cases.

The work presented in this chapter points to several avenues of further research. For instance, I would like to explore alternative ways to model ALR, including, for example, similarity-weighted versions of other statistics, such as the median or the mode. As mentioned in Section 3.4, I also would like to investigate a hybrid

[45] As noted by Hunter (1998), a number of accounts of ALR conflate or equate analogy and explicit induction (e.g., Levi 1949; Posner 1990, 1995, 2008). In other accounts of legal reasoning, the relation between analogy and explicit induction (as well as deduction and abduction) are more nuanced (e.g., Sunstein 1993; Brewer 1996; Sunstein 1996).

[46] Recall the discussion on the connection between empirical similarity theory and kernel regression in Section 3.3.1.

model along the lines of a mixed SAR model. In addition, I would like to probe the extent to which non-legalist theories of judicial behavior could be formalized using statistical models. For example, I believe one could profitably model an attitudinalist judge as a Bayesian nonparametric statistician. Finally, I would like to examine areas of law other than maritime salvage. Although this likely would require further data collection on my part, one area in which potentially suitable data already have been collected is U.S. criminal confession cases.[47]

[47] See Benesh (2002) and Kastellec (forthcoming).

APPENDIX

OVERVIEW OF U.S. MARITIME SALVAGE LAW

Throughout the colonial period, royal English courts decided salvage cases (Mangone 1997). Following independence, the U.S. Constitution granted federal courts original jurisdiction in "all Cases of admiralty and maritime jurisdiction,"[48] which include salvage cases. By the end of the nineteenth century, most salvage law concepts were generally settled (Mangone 1997). Although the United States is a party to both the 1910 Brussels Salvage Convention[49] and the 1989 London Salvage Convention,[50] "U.S. courts usually decide salvage controversies under the principles of the general maritime law without reference to international conventions" (Force 2004; see also Gilmore and Black 1975).[51]

Jurisdiction and Types of Actions

By statute, subject matter jurisdiction to adjudicate salvage claims lies with the federal courts.[52] When the matter at controversy is whether salvage is due and, if due, the amount, a federal court applying admiralty law is the only one in which

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[48] U.S. Const. art. III, § 2.

[49] International Convention for the Unification of Certain Rules Relating to the Salvage of Vessels at Sea (1910). Congress codified the 1910 Convention, with minor changes, as the Salvage Act of 1912 (Gilmore and Black 1975; Mangone 1997).

[50] International Convention on Salvage (1989).

[51] See, e.g., *Sobonis v. Steam Tanker Nat'l Defender*, 298 F. Supp. 631 (S.D.N.Y. 1969) (allowing salvage awards without reference to the Salvage Treaty).

[52] "The district courts shall have original jurisdiction, exclusive of the courts of the States, of:

(1) Any civil case of admiralty or maritime jurisdiction, saving to suitors in all cases all other remedies to which they are otherwise entitled.

(2) Any prize brought into the United States and all proceedings for the condemnation of property taken as prize."

28 U.S.C. § 1333. The Supreme Court assumed that salvage claims were within federal admiralty jurisdiction as early as 1804. See *Mason v. The Blaireau*, 6 U.S. (2 Cranch) 240 (1804); *Treasure Salvors, Inc. v. Unidentified, Wrecked and Abandoned Sailing Vessel*, 640 F.2d 560 (5th Cir. 1981) ("Claims arising out of salvage operation—efforts to rescue or recover ships disabled or abandoned at sea or to retrieve their cargo—are, unquestionably, within the admiralty jurisdiction of the federal courts.").

such questions can be tried.[53] By contrast, an action based on quantum meruit, or for a case where a contract has been formed, may be tried in a state court, which has the authority to "assess damages based upon contract but cannot make a salvage award" (Norris 2008). A salvage suit is generally brought in rem against a ship, its cargo, or both because salvors, under maritime law, have a lien upon the property salved; if the ship or cargo is unavailable—due to destruction or removal from the jurisdiction—a salvor may seek remedy from the owner directly or in personam.[54] Under current rules, a suit in rem and in personam may be joined.[55] The federal court of appeals, when hearing an appeal of a federal district court's salvage award, "will not disturb a salvage award unless it is based on erroneous principles or a misapprehension of the facts or is so grossly excessive or inadequate as to be deemed an abuse of discretion."[56]

Types of Salvage Services

Salvage services fall into two categories: "contract" salvage and "pure" salvage.

Contract salvage occurs when the owner of property enters into an agreement with a salvor to rescue imperiled assets (Norris 2008). A salvage contract may be entered into before any emergency or after the ship or cargo is already in peril (Mangone 1997). The most common contract of this sort is the Lloyd's of London Open Form (LOF), although there is no obligation on any party to utilize this document to form a valid contract (Mangone 1997).[57] Although a court will

[53] *Houseman v. The North Carolina*, 40 U.S. (15 Pet.) 40 (1841).

[54] *The Sabine*, 101 U.S. 384 (1879).

[55] *The G.L. 40*, 66 F.2d 764 (2d Cir. 1933).

[56] *Compania Galeana, S.A. v. Motor Vessel Caribbean Mara*, 565 F.2d 358 (5th Cir. 1978). See also *Oelwerke Teutonia v. Erlanger & Galinger*, 248 U.S. 521 (1919) ("Unless there has been some violation of principle or clear mistake, appeals to this Court concerning the amount of the allowance are not encouraged.").

[57] The LOF is four pages long, does not specify any sums, and proclaims its fundamental premise of "no cure-no pay" in large, bold letters. The LOF also contains provisions relating to enforcement of the contract through arbitration in London—a provision that U.S. courts have declined to enforce when the clause of the LOF is the only connection to an otherwise domestic

closely examine a contract to make sure that neither side has taken advantage of an emergency to subject the other party to "grossly unfair terms" (Mangone 1997),[58] a contract wherein an owner has struck a "hard bargain" or where "the service was attended with greater or less difficulty than was anticipated, will not justify setting [the contract] aside."[59] Should a contract be thrown out, however, due to misconduct on the part of a shipowner or captain, innocent crew members may still be entitled to an award.[60]

Pure salvage is rendered voluntarily in the absence of a contract. The reward for a person at sea who rushes to save another's property is "generously computed" as a matter of public policy (Gilmore and Black 1975). Because a salvor may never claim title to property by salvaging it, the need for incentive to save the property of another evolved into a salvage award (Mangone 1997).

Three elements are necessary for a valid pure salvage claim: (1) a marine peril; (2) service voluntarily rendered when not required as an existing duty or from a special contract; and (3) success in whole or in part, or that the service rendered contributed to such success.[61] The peril involved in a salvage operation does not necessarily have to be immediately impending—just subjecting the ship to the potential danger of damage or destruction.[62]

Because they already have a duty to aid their own ship in peril, crew members

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operation between U.S. citizens. *Jones v. Sea Tow Services Freeport NY Inc.*, 30 F.3d 360 (2d Cir. 1994).

[58] This cuts both ways—the salvors could, of course, extort a favorable agreement, but the party in peril could also conceal the extent of danger or damage to its own advantage (Gilmore and Black 1975).

[59] *The Elfrida*, 172 U.S. 186 (1898).

[60] *Jackson Marine Corp. v. Blue Fox*, 845 F.2d 1307 (5th Cir. 1988).

[61] *The Sabine*, 101 U.S. 384 (1879). Note the difference between simple "towage," which is for the simple convenience of another vessel in expediting her passage, and "salvage," which also includes an element of peril. *McConnochie v. Kerr*, 9 F. 50 (S.D.N.Y. 1881).

[62] *Fort Myers Shell & Dredging Co. v. Barge NBC 512*, 404 F.2d 137 (5th Cir. 1968). In the spectrum of peril, the degree of danger is immaterial—the degree "can affect the amount of the award, but not the establishment of a salvage service" (Norris 2008).

may not be considered "voluntary" salvors of their own ship[63]—nor may any person engaged in a profession which creates such a duty to salvage, such as firemen (Force 2004).[64] Regardless of heroic or costly measures to salvage property, any attempt that results in the complete loss of property will not be rewarded—nor will costs be reimbursed (Norris 2008).[65] A party may not force its services upon any owner or master of a vessel who does not want assistance.[66]

The Supreme Court has made clear that only maritime property can be salvaged,[67] however the definition of maritime property can include any vehicle, cargo, or object with a nexus to traditional maritime activities.[68] The saving of life, in

[63] They may, however, participate in salvage operations for reward if "their ship has been abandoned without hope of recovery, or the crew has been legally discharged from further services by the master." *Drevas v. U.S.*, 58 F. Supp. 1008 (D. Md. 1945). Common ownership of the salving vessel and the salved vessel does not necessarily preclude salvors from claiming award. 46 U.S.C § 80107(b). In contrast, passengers have "no duty to a vessel or its cargo" and cannot be compelled to assist in saving either (Mangone 1997). For this reason, passengers may "for extraordinary services, and the use of extraordinary means, not furnished by the equipment of the ship herself, by which she is saved from imminent danger . . . have salvage." *The Connemara*, 108 U.S. 352 (1883).

[64] *In re Iowa Fleeting Service, Inc.*, 211 F. Supp. 2d 794(M.D. La. 2002) ("Firefighters are precluded from obtaining a salvage award when the salvage work they perform is in the course of their existing duties as firefighters."). But see *Markakis v. S/S Volendam*, 486 F. Supp. 1103 (S.D.N.Y. 1980) (noting that even within certain professions, actions outside the line of duty may entitle a salvor to an award). Note, however, that nothing legally precludes the U.S. government from claiming salvage, even though it usually does not as a matter of policy (Gilmore and Black 1975).

[65] *Scott v. The Clara E. Bergen*, 21 F. Cas. 816 (D.S.C. 1882) (No. 12526a) ("All attempts, however costly, meritorious, or praiseworthy, go for nothing. In the event of failure, [the salvor] has to make his own repairs and pocket all losses, and he must give before he can get. He must save before he can ask to share what is saved. The owner, in fact and in law, can only be called upon to give to the salvor a portion of that very property which the salvor has saved for him; to restore only a portion of that which, but for the salvor, would have been lost to him. Thus it is the salvor who enables the owner to make the payment.").

[66] *The Indian*, 159 F. 20 (5th Cir. 1908). In contrast, salvors may start operations on an abandoned vessel found at sea without prior authorization—with the hope of later reward (Mangone 1997).

[67] *Cope v. Vallette Dry-Dock Co.*, 119 U.S. 625 (1887) ("[N]o structure that is not a ship or vessel is a subject of salvage.").

[68] *Broere v. Two Thousand One Hundred Thirty-Three Dollars*, 72 F.Supp. 115 (1947) (finding that that money found on a human body floating on navigable waters was a proper subject of salvage); *Lambros Seaplane Base v. The Batory*, 215 F.2d 228 (2d Cir. 1954) (considering a seaplane which crashed in navigable waters a proper subject of salvage). But see *Provost v. Huber*, 594 F.2d 717 (8th Cir. 1979) (holding that house being dragged across a frozen lake had not embarked upon a "maritime adventure" and dismissing the salvage action for lack of a nexus with traditional maritime activities). Property considered salvable also traditionally includes any

contrast to property, does not on its own confer an award of salvage (Gilmore and Black 1975).[69] To discourage disregard for human life in favor of saving property in times of emergency, however, U.S. law allows for salvors of human life to receive a "fair share" of the salvage award.[70] In contrast to the law of salvage, the law of "finds" controls property for which the title has been irrevocably lost and usually applies to ancient shipwrecks (Schoenbaum 2004). Historically, the court might award the title in place of a salvage award for property for which no owner came forward, but the Abandoned Shipwreck Act of 1987 transferred ownership of most such property to the government (Norris 2008).

Salvage Awards

In *The Blackwall*, 77 U.S. (10 Wall.) 1 (1869), the Supreme Court first listed the six factors for the determination of a salvage award: (1) the labor expended by the salvors in rendering the salvage service; (2) the promptitude, skill, and energy displayed in rendering the service and saving the property; (3) the value of the property employed by the salvors in rendering the service, and the danger to which such property was exposed; (4) the risk incurred by the salvors in securing the property from the impending peril; (5) the value of the property saved; and (6) the degree of danger from which the property was rescued. The value of any potential cargo of the salving ship is not a factor (Norris 2008).[71] By contrast, the value of freight and cargo of the salved ship are taken into account when computing the value of the property saved (Schoenbaum 2004). The burden of establishing the

object which has been "thrown overboard" (jetsam), "found freely floating on the sea" (flotsam), "attached to buoys" (ligan), and "washed up to shore from the sea" (lagan) (Mangone 1997).

[69] Note that masters of ships at sea are obliged by statute to "render assistance to any individual found at sea in danger of being lost, so far as the master or individual in charge can do so without serious danger to the master's or individual's vessel or individuals on board." 46 U.S.C. § 2304(a)(1).

[70] 46 U.S.C. § 80107(a).

[71] See, e.g., *The Ereza*, 124 F. 659 (E.D. Pa. 1903).

value of the salved property is on those seeking the award and should be assessed according to fair market value.[72]

There is no "precise formula" for computing salvage awards.[73] Although all of the factors should be considered in determining the award, each factor is not given equal weight (Force 2004). The trial court has considerable discretion in weighing the factors and making its determination on a case-by-case basis.[74] Courts do not view salvage awards "merely as pay, on the principal of a quantum meruit, or as a remuneration pro opere et labore, but as a reward given for perilous services, voluntarily rendered, and as an inducement to seamen and others to embark in such undertakings to save life and property."[75] Salvage awards are apportioned among co-salvors according to the "relative participation and risk" of each salvor (Norris 2008).[76] The statute of limitations for securing a salvage award is two years,[77] and the owners of both the ship and its cargo are liable to pay.[78]

As noted above, the salvage must be successful to merit an award. However, the actions of a salvor that worsen the position of the salved property may reduce the award, preclude it, or result in an award of damages (Force 2004). If the salvage is successful, but the operation causes some damage through ordinary negligence,

[72] *Nolan v. A. H. Basse Rederiaktieselskab*, 267 F.2d 584 (3d Cir. 1959).

[73] *Allseas Maritime, S.A. v. M/V Mimosa*, 812 F.2d 243 (1987).

[74] *The Emulous*, 8 F. Cas. 704 (C.C.D. Mass.) (No. 4480) ("And here, again, the court is asked to lay down some rules, by which to guide the parties in interest, underwriters as well as owners, in the ascertainment of the proper rate of salvage. That is asking the court to do, what it is utterly impracticable to do, to lay down rules, in cases admitting of an indefinite diversity of circumstances, and endless considerations of value, of perils, of services, and of merit. The subject is necessarily one, in which the reward must depend upon a just estimate of all the circumstances of each particular case."); *The Rescue v. the George B. Roberts*, 64 F. 139 (E.D. Pa. 1894) ("At best the award must be the result of an intelligent guess.").

[75] *The Blackwall*, 77 U.S. (10 Wall.) 1 (1870). See also *B.V. Bureau Wijsmuller v. United States*, 702 F.2d 333 (2d Cir. 1983).

[76] See, e.g., *The Lydia*, 49 F. 666 (E.D.N.Y. 1892).

[77] 46 U.S.C. § 80107(c).

[78] In general, the owner of the cargo will be responsible for its own portion of the award and the owner of the ship for its portion; however, the court has discretion to apportion the award and the two owners may make other agreements between themselves before disbursement of the cargo (Schoenbaum 2004; Norris 2008).

the salvor is liable and the court will be reduce the award accordingly; if, however, the salvor causes damage through "gross negligence or willful misconduct," then the court may deny an award or even award affirmative damages.[79] Fraud or other dishonest conduct also may deprive a salvor of an award (Mangone 1997).

[79] *Basic Boats, Inc. v. U.S.*, 352 F. Supp. 44 (E.D. Va. 1972); Schoenbaum (2004). Note that professional salvors are held to a higher standard of care than non-professionals (Mangone 1997).

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